Evaluating the Performance of Croon's Method of Moments Corrected Maximum Likelihood Estimator with Cluster-randomized Studies Probing Mediation

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Purpose

The purpose of this study was to evaluate the performance Croon's method of moments corrected maximum likelihood estimator with cluster-randomized studies probing mediation. Cluster-randomized studies or trials (CRTs) aimed at mediated effects have proven effective in operationalizing complex theories of action common in educational research. However, these studies are critically limited when they disregard the potentially deleterious effects of measurement error. More specifically, most studies in education are conducted using latent variables that are subject to measurement error while analyses are conducted without accounting for the possibility of measurement error. A multilevel structural equation modeling framework estimated using full information maximum likelihood has been shown to be an effective strategy to address the deleterious effects of measurement error (e.g., bias) but often requires prohibitively large sample sizes when utilized in planned educational research (e.g., Schochet, 2011; Spybrook, Shi, and Kelcey, 2016). We conducted a simulation study to outline the performance Croon's corrected maximum likelihood estimator with cluster-randomized studies probing mediation. Results indicate the effectiveness and feasibility of the estimator as an alternative approach that properly considers measurement error and is suitable for sample sizes common in planned educational research.

Background

In educational research, it is extremely common to conduct investigations in hierarchical settings (e.g., students nested within schools) involving variables with complex theories of action that are not directly observed and subject to measurement error. Multilevel structural equation models are suitable for these conditions but require samples sizes exceeding feasible limits of planned educational research. Croon's corrected maximum likelihood estimator accurately estimates parameters in cluster-randomized studies aimed at mediated effects (Kelcey, Cox, and Dong, in press) and in single-level sequential mediation models with increasingly complex measurement models (Kelcey, 2019) but its performance in CRTs aimed at mediated effects with limited sample sizes typical in educational research is unclear.

Method

There are four general steps to Croon's method of moments corrected maximum likelihood estimator (Croon, 2002): (a) estimate scores for each latent construct; (b) estimate the covariance matrix of the scores; (c) correct the covariance matrix using results from the measurement models estimated in step (a); and (d) estimate a bias-corrected path model using the corrected covariance matrix from step (c). The core of Croon's corrected ML estimation is the correction in step (c). This process allows unbiased parameter estimates of mediated effects but with smaller sample sizes because it is a limited information approach.

To evaluate the performance of Croon's corrected ML estimation under small sample size conditions, we conducted a simulation study using a structural equation model formulation of 2-1-1 mediation with common factor models for the covariates (*W* and *X*), the mediator (M), and the outcome (*Y*) and a structural model to connect them (e.g., Preacher, Zyphur, & Zhang, 2010). This model is illustrated in Figure 1 in which a treatment (T) is assigned at the cluster-level and influences an individual-level outcome (Y), through an individual-level mediator (M). We include covariates at the cluster- (W) and individual-level (X).

Simulation

We applied Croon's corrected ML estimation, uncorrected factor score path analysis (FS), and maximum likelihood estimation to the CRT represented Figure 1 across a 1000 data sets and 720 conditions. Sample size conditions include individual per cluster sample sizes of $n_1 = 80, 40, 20, 10, 5$ and cluster sample sizes of $n_2 = 100, 80, 50, 30, 20, 10$. We also considered different intraclass correlation coefficients, indicators per latent variable, and indicator weights. The conditions represent a variety of real-world scenarios and include the factors likely to influence parameter estimation in CRTs probing mediation.

Results

Convergence rate. We tracked the failure of an estimation method to provide a solution across all conditions. Convergence failure rates increased as sample sizes decreased for all three estimators (see Figure 2). In almost every condition, Croon's corrected ML estimation had the smallest convergence failure rate followed closely by the FS approach with maximum likelihood estimation often incurring the largest convergence failure rates.

Bias. For bias, we calculated the average absolute bias for each estimation approach as the averaged difference between the estimated value and the true coefficient across the 1000 data sets. We found less bias across approaches with larger sample sizes with varying results depending on indicator weights and estimation approach. Bias for each estimator actually held relatively steady when cluster sample size was greater than 50 with Croon's corrected ML estimation and maximum likelihood performing similarly and the FS approach typically incurring the most bias (see Figure 2). Once again, we found smaller indicator weights led to an increase in bias for each approach (see Figure 3). Notably, Croon's corrected ML estimation did outperform maximum likelihood in many small sample size conditions.

Error variance. The final criterion of interest was the standard deviation (*SD*) of path coefficient estimates. The *SD* results by approach provide some context to the results involving bias so we scaled the size of the symbols representing bias in Figures 2 and 3 based on the average *SD* of the estimates. While maximum likelihood performed relatively well in terms of bias, it consistently had the largest *SD*. Conversely, the FS approach incurred the most bias but consistently had the smallest *SD*. Croon's corrected ML estimation approach tended to balance these considerations with relatively low bias and *SD* well below the maximum likelihood approach but greater than typically achieved using the FS approach.

Significance

The results imply Croon's corrected ML estimation is an effective approach with cluster-randomized studies probing mediation. Through increased reliability, efficiency, and reductions in bias, Croon's corrected ML estimation increases the feasibly of planned multilevel studies in education under common conditions (e.g., limited sample sizes and complex theories of action). Increases in feasibility are mainly achieved through the estimator's ability to reduce the scale of studies necessary to examine complex theories involving latent constructs in hierarchically structured settings. Put differently, using Croon's corrected ML estimation allows educational researchers to examine complex theories with smaller cluster and individual per cluster sample sizes.

References

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Figure 1

Conceptual representation of a 2-1-1 mediation model from a two-level cluster randomized trial in which the mediator, outcome, and covariates are subject to measurement error.

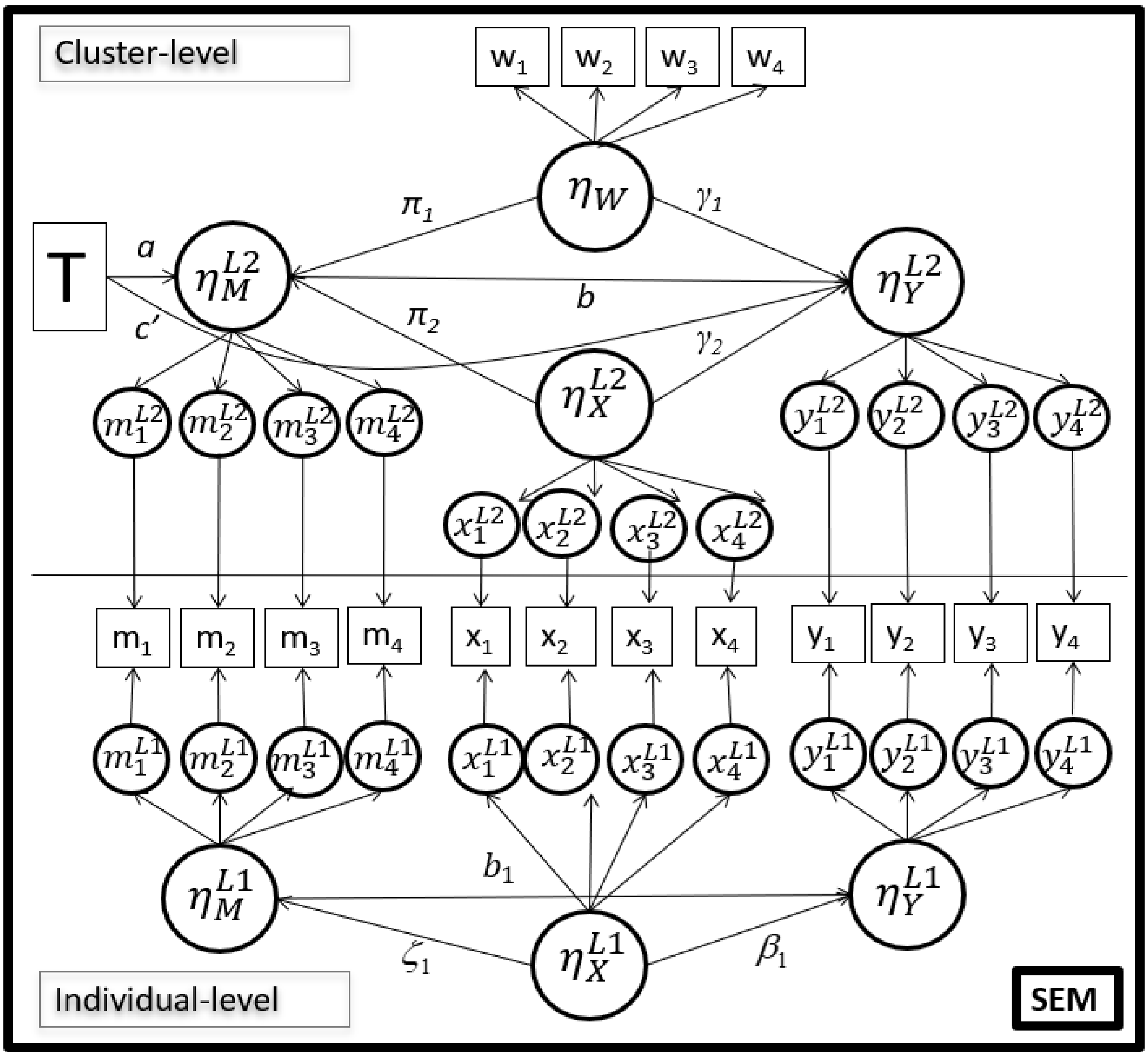
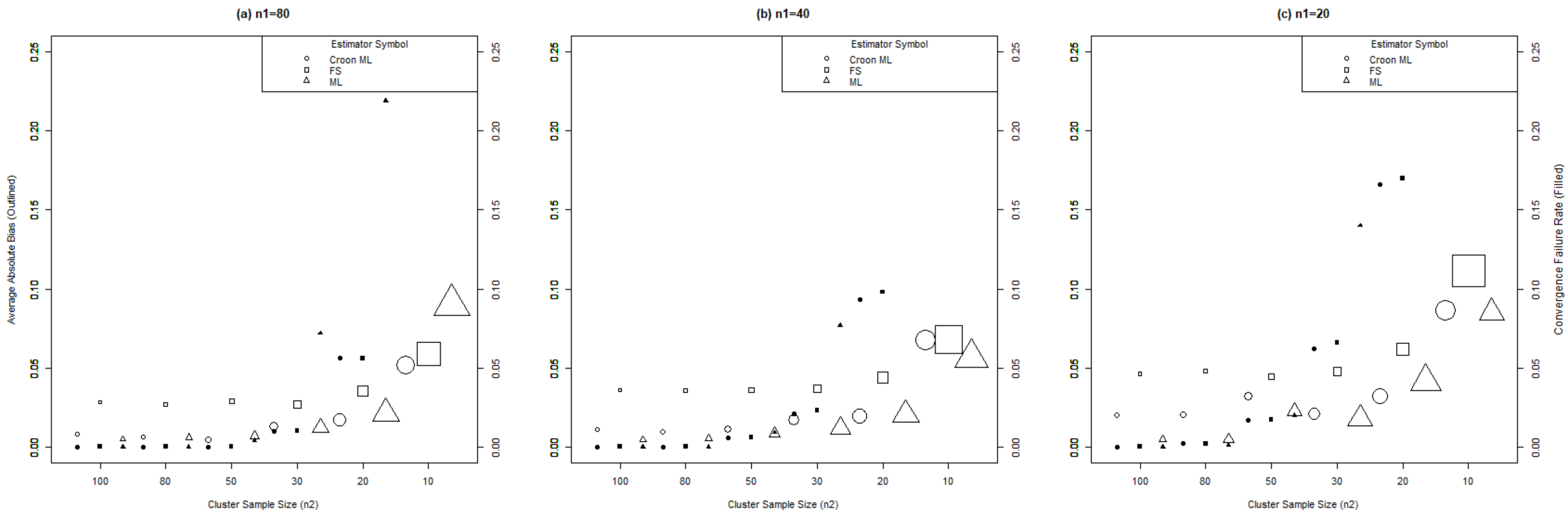
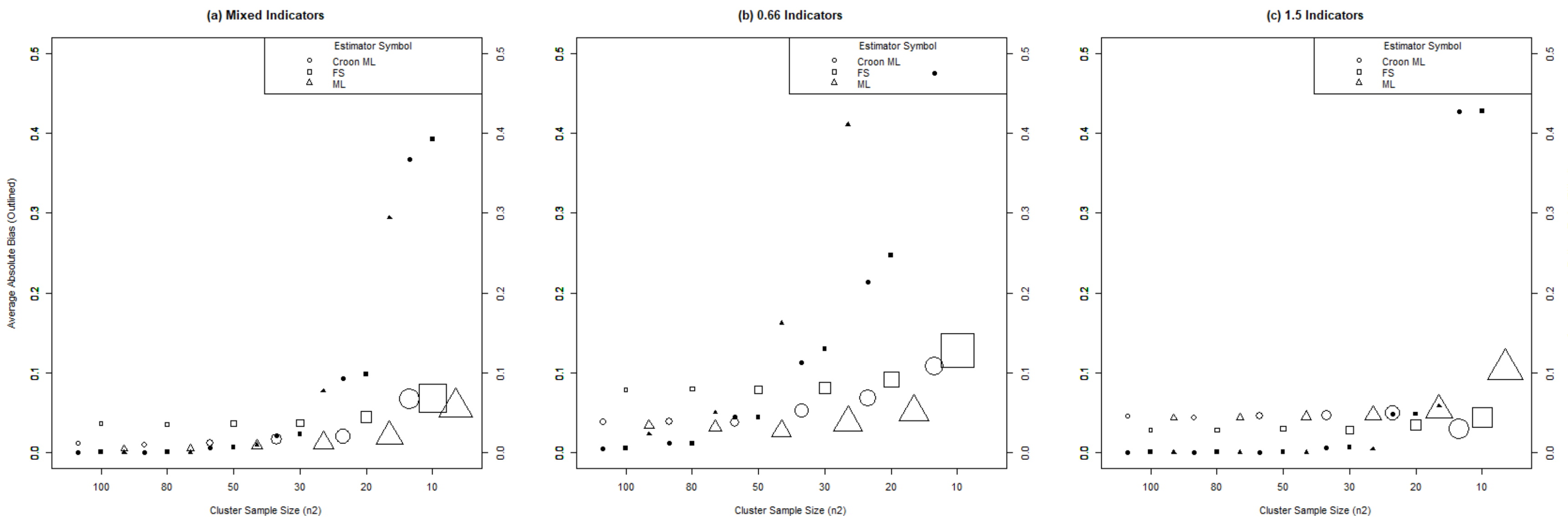


Figure 2 Bias (with size based on *SD*) and Convergence Rate by Cluster and Individual per cluster Sample Size for each Estimator with different n_1 values in each n_2



Note. The size of each point indicating average absolute bias (outlined) reflects the average SD of the coefficient estimates for that estimator under the model and conditions indicated. A larger SD results in larger points on the plot with smaller SDs leading to smaller points.

Figure 3 Bias (with size based on *SD*) and Convergence Rate by Cluster Sample Size for each Estimator with different indicator weights



Note. n1=40; the size of each point indicating average absolute bias (outlined) reflects the average SD of the coefficient estimates for that estimator under the model and conditions indicated. A larger SD results in larger points on the plot with smaller SDs leading to smaller points.