# Power Analysis for Moderator Effects in Longitudinal Cluster Experimental Designs

**Authors and Affiliations:**

Wei Li
College of Education
University of Alabama
307C Carmichael Hall, Tuscaloosa, AL 35487
Phone: (205) 348-8087
Email: wli59@ua.edu

Spyros Konstantopoulos
College of Education
Michigan State University
Room 4, Erickson Hall, East Lansing, MI 48824
Email: spyros@msu.edu

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(998 words)

**Theoretical Framework**

Frequently Cluster randomized trials (CRTs) in education entail a longitudinal component where for example students are followed across grades. Suppose a researcher is interested in estimating the change in mathematics for students who use a novel mathematics curriculum vis-a-vis students who use a traditional mathematics curriculum.

Trajectories of change can be represented via polynomial functions (linear, quadratic, etc.) (Raudenbush & Liu, 2001). Studies about polynomial change may have a nested structure. In education, for example, repeated measurements of student achievement are nested within students, who are nested within schools. Previous work has used two-level (e.g., measurements within students) and three-level models (e.g., measurement within students, and students within schools) to compute power to detect treatment effects in longitudinal CRTs (Li & Konstantopoulos, 2017, 2019; Raudenbush & Liu, 2001).

However, frequently the objective is to determine whether treatment effects on student achievement vary across student (e.g., gender, ethnicity, or SES) or school (e.g., rural versus urban) characteristics. These student and school variable/characteristics are called moderators. A moderator effect is represented in linear models via an interaction term, and its significance indicates the treatment effect is influenced by the moderator. Prior work has provided formulas for power analyses of moderator effects in cross-sectional two- and three-level CRTs (Dong, Spybrook, & Kelcey, 2018; Spybrook, Kelcey, & Dong, 2016).

**Purpose**

Currently, there are no studies that provide power formulas for moderator effects in longitudinal two- and three-level CRTs. This study fills in the literature gap and provides methods for power analysis of tests of moderator effects in longitudinal CRT designs. We discuss two- (e.g., repeated measures nested within students) and three-level (e.g., repeated measures nested within students nested within schools) designs where the treatment is at the second or third level respectively. Because of space limitations we only focus on three-level designs.

**Methods**

We utilized orthogonal polynomial contrast coefficients to describe our models below because they are independent of each other.

Three-Level Model with a Level-3 Moderator

Consider a three-level longitudinal CRT design where level-3 units (e.g., schools) are randomly assigned to treatment or control conditions. Suppose there is only one level-3 moderator; then the first, second and third-level models are expressed as

Level 1: $Y_{gij} = \alpha_{0ij}c_{0g} + \alpha_{1ij}c_{1g} + \alpha_{2ij}c_{2g} + \cdots + \alpha_{(K-1)ij}c_{(K-1)g} + e_{gij}, e_{gij} \sim N(0, \sigma_e^2),$

Level 2: $\alpha_{kij} = \beta_{k0j} + \boldsymbol{B_{k1j}X_{ij}} + \xi_{kij}, \xi_{kij} \sim N(0, \tau_{k|\mathbf{X}}^2),$ (1)

Level 3: $\beta_{k0j} = \gamma_{k00} + \gamma_{k01}T_j + \gamma_{k02}S_j + \gamma_{k03}(S_jT_j) + \boldsymbol{\Gamma_{k04}Z_j} + u_{k0j}, u_{k0j} \sim N(0, \omega_{0k|TS\mathbf{Z}}^2),$

where $c_{kg}$ represent orthogonal polynomial contrasts of degree *k* (*k* = 0, 1, …, *K*-1) at measurement occasion *g* (*g* = 1, …, *G*), $\alpha_{pi}$'s represent the mean and the rates of change (linear, quadratic, cubic, etc.), $e_{gi}$ is level-1 residual with variance $\sigma_e^2$, $\boldsymbol{X_{ij}}$ is a row vector of *q* level-2 covariates, $\xi_{kij}$ is a level-2 residual with variance $\tau_{k|\mathbf{X}}^2$, $S_j$ is a level-3 continuous moderator, $\boldsymbol{Z_j}$ is a row vector of *v* level-3 covariates, $\gamma_{k03}$ is the moderator effect, and $\eta_{k0j}$ is a level-3 residual with variance $\omega_{0k|TS\mathbf{Z}}^2$. The variance of $\hat{\beta}_{k0j}$ is

$$Var(\hat{\beta}_{k0j}) = \eta_{3k}\omega_{0k}^2 + (\eta_{2k}\tau_k^2 + \sigma_k^2)/n, \tag{2}$$

where $\omega_{0k}^2$ is the level-3variance for the k-*th* polynomial change in the unconditional model; $\tau_k^2$ is the level-2 variance for the *k*-th polynomial change in the unconditional model; $\eta_{3k} = \frac{\omega_{0k|TS\mathbf{Z}}^2}{\omega_{0k}^2}$, is the proportion of unexplained variance at level-3; $\eta_{2k} = \frac{\tau_{k|\mathbf{X}}^2}{\tau_k^2}$, is the proportion of unexplained variance at level-2; $\sigma_k^2 = Var(\hat{\alpha}_{ki}|\alpha_{ki}) = \frac{\sigma_e^2}{\sum_{g=1}^{G} c^2{}_{kg}}$, is the level-1 variance of the *k*-th polynomial change, $\sum_{g=1}^{G} c^2{}_{kg} = k_k^2 \cdot \frac{(k!)^4}{(2k)!\cdot(2k+1)!} \cdot \frac{(G+k)!}{(G-k-1)\ !}$, and $k_k$ is a positive constant.

Suppose there are *M* schools and within each school there are *n* students. The variance of the moderator effect is

$$Var(\hat{\gamma}_{k03}) = \frac{Var(\hat{\beta}_{k0j})}{P(1-P)M\zeta_S^2} = \frac{\eta_{3k}\omega_{0k}^2 + (\eta_{2k}\tau_k^2 + \sigma_k^2)/n}{P(1-P)M\zeta_S^2}, \tag{3}$$

where $\zeta_S^2$ is the variance of the level-3 moderator $S_j$.

Suppose that $\tau_k^2 + \sigma_k^2 = 1$, $\zeta_S^2 = 1$ and the effect size is $\delta_k = \hat{\gamma}_{k03}\sqrt{\frac{\zeta_S^2}{\omega_{0k}^2 + \tau_k^2}}$, where $\omega_{0k}^2$, $\tau_k^2$, and $\sigma_k^2$ are level-3, level-2, and level-1 variance components respectively in the unconditional model. Then, the standardized formula of the non-centrality parameter of the *t*-test of the moderator effect is

$$\lambda = \delta_k\sqrt{\frac{P(1-P)Mnr_k}{n\eta_{3k}\rho_k r_k + [1-(1-\eta_{2k})r_k)](1-\rho_k)}}, \tag{4}$$

where $r_k = \frac{\tau_k^2}{\tau_k^2 + \sigma_k^2}$, is the reliability of the least-squares estimator $\hat{\alpha}_{kij}$, and $\rho_k = \frac{\omega_{0k}^2}{\omega_{0k}^2 + \tau_k^2}$ is the intraclass correlation coefficient (ICC) for the *k*-th polynomial change. The degrees of freedom (*df*) of the *t*-test are $df = M - v - 4$, when the model includes three terms (the treatment variable, the moderator, and the interaction term between the two), one intercept, and *v* level-3 covariates.

When $S_j$ is a binary moderator (e.g., rural versus urban schools) $\zeta_w^2 = Q(1-Q)$, and $Q$ is the proportion of rural schools. The variance of $\hat{\gamma}_{k03}$ becomes

$$Var(\hat{\gamma}_{k03}) = \frac{\eta_{3k}\omega_{0k}^2 + (\eta_{2k}\tau_k^2 + \sigma_k^2)/n}{P(1-P)Q(1-Q)M}, \tag{5}$$

and the non-centrality parameter of the *t*-test becomes

$$\lambda = \delta_k \sqrt{\frac{P(1-P)Q(1-Q)Mnr_k}{n\,\eta_{3k}\rho_k r_k + [1-(1-\eta_{2k})r_k)](1-\rho_k)}}, \quad (6)$$

where $\delta_k = \hat{\gamma}_{k03}\sqrt{\frac{1}{\omega_{0k}^2+\tau_k^2}}$. The *df* of the t-test are $df = M - v - 4$. The power of a two-tailed *t*-test for a specified significance level α is

$$p = 1 - H\,[c(\alpha/2, df), df, \lambda] + H\,[-c(\alpha/2, df), df, \lambda], \quad (7)$$

where c indicates a critical value and H the cumulative distribution function.

**Illustrative Example**

To demonstrate the applicability of the methods we utilize an example of a three-level longitudinal design with a binary moderator at the top level.

Suppose a researcher wants to examine whether the effects of a school assessment program on students' reading achievement are moderated by school urbanicity (rural versus urban schools). Suppose this is a longitudinal three-level CRT design (e.g., measurements nested within students nested within schools) where schools are randomly assigned to treatment or control conditions and student measurements are collected for four years. A three-level model is used to evaluate whether the effect of the assessment program on the linear rate of change in reading achievement differs between rural and urban schools. The treatment, the moderator, their interaction, and one covariate are included in the third level.

Suppose the design is balanced ($P$ = 0.5) and there are 40 schools and within each school 20 students. Following prior work (Li & Konstantopoulos, 2017), suppose the reliability and ICC are $r_1 = 0.664$, $\rho_1 = 0.117$ respectively. Also, suppose $\eta_{31} = \eta_{21} = 0.500$, $\delta_1 = 0.400$, half of the schools are rural schools ($Q$ = 0.5), and the significant level is set at 0.05. Then, the non-centrality parameter of a two-tailed *t*-test using equation (6) is

$$\lambda = \delta_1 \sqrt{\frac{P(1-P)Q(1-Q)Mnr_1}{n\eta_{31}\rho_1 r_1 + [1-(1-\eta_{21})r_1](1-\rho_1)}} = 0.4 \times \sqrt{\frac{0.5\times(1-0.5)\times0.5\times(1-0.5)\times40\times20\times0.505}{20\times0.5\times0.117\times0.664+[1-(1-0.5)\times0.664](1-0.117)}} = 1.971.$$

The critical value of the t-test with $df = 40 - 5 = 35$ is $c$(0.05, 35) = 2.030 and the power of the moderator effect is

$$p = 1 - H\,[2.030, 35, 1.971] + H\,[-2.030, 35, 1.971] = 0.483.$$

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