On the Robustness of Doubly Robust Estimators in Causal Inference

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1 Abstract

Doubly robust (DR) estimators that combine regression adjustments and inverse probability weighting (IPW) are widely used in causal inference with observational data because they are claimed to be consistent when either the outcome or the treatment selection model is correctly specified (Scharfstein et al., 1999). This property of *double robustness* is desirable in educational research because researchers often lack strong subject matter theory for choosing the correct functional form of the relationship between the outcome and the covariates or between the propensity score and the covariates. In this case, DR estimators provide additional protection and thus help reduce confounding bias. In the original papers on DR estimators, the same set of covariates is used in both models to prove the double robustness property. However, in practice empirical researchers often use different covariate sets for the two models because they select the covariates for each model separately, either using subject matter theory or statistical techniques like stepwise regression (Lee et al., 2011; Wilson and Chermak, 2011). In such a situation, the conditions for DR estimators to be consistent become more complex. Even with two correctly specified models, the DR estimator can still be biased and inconsistent, but this issue has not been noticed.

We demonstrate that the consistency of DR estimators relies on an implicit condition: the covariates in the outcome model and the selection model must be identical. That is, DR estimators guarantee protection against incorrectly specified functional forms given a set of covariates, but they do not guarantee robustness to deliberate variable selection. This issue is important because many researchers, even some methodologists, are not aware of it. Some methodological papers demonstrate the properties and implementation of DR estimators through examples or simulations, but with different sets of covariates for the

outcome and selection model, resulting in misleading guidance because double robustness is no longer guaranteed (Glynn and Quinn, 2010; Funk et al., 2011; SAS Institute Inc, 2017). It is also worth noting that DR estimators have already been implemented in several statistical packages, but they usually allow users to specify different covariate sets for the two models (Funk et al., 2007; Emsley et al., 2008; Zetterqvist and Sjölander, 2015; SAS Institute Inc, 2017). Users should be warned that they must include the same covariates in both models to guarantee the double robustness property.

This paper reviews two classes of commonly used DR estimators. (1) The augmented inverse probability weighted (AIPW) estimator, relies on independently estimated outcome and selection models (Bang and Robins, 2005); (2) The inverse probability weighted regression (IPWR) estimator, uses inverse-probability-of-treatment weights to fit the outcome model (Schafer and Kang, 2008). We use both formal theoretical derivations and simulation results to demonstrate under which conditions double robustness breaks down despite a correctly specified outcome or selection model.

Then, we provide sufficient conditions for DR estimators to be consistent when the two models use different covariates (see Table 1 for the formal results). These conditions are: (a) the joint set of covariates used in the outcome and selection model satisfies the strong ignorability assumption, and (b) at least one of the two models is correctly specified with respect to *all* covariates in the joint covariate set. The key is that the covariates dropped from the bias-removing outcome model must not be predictive of the outcome, or the covariates dropped from the bias-removing propensity score model must not be predictive of treatment selection. Importantly, the condition is only about prediction and not about causal relations. However, as already mentioned, this complexity can be avoided by using the same set of covariates in both models.

To illustrate these results in a more intuitive way, we present simulation results from a simple data generating process (Figure 1) in Table 2 and Table 3. Since both functional forms are correctly specified, any remaining bias is due to variable selection. DR estimators can be inconsistent even though both the outcome and selection model may remove the entire bias on their own, as shown in the last column of Table 2. Table 3 shows that collider bias produces inconsistent estimates even when one model is correct.

To summarize, using different covariate sets in DR estimators may cause serious problems because the outcome and selection model interfere with each other.

Nonetheless, it is good practice to use DR estimators for causal inference in educational research for they provide protection against incorrectly specified functional forms, but researchers should carefully choose a single covariate set for both the outcome and the selection model.

2 Tables

Table 1: Sufficient conditions for consistency. $\boldsymbol{A}$ and $\boldsymbol{B}$ are covariate sets for the outcome and the propensity score respectively, Y^0 and Y^1 are potential outcomes, Z is the treatment indicator, $m^z(\cdot, \hat{\boldsymbol{\alpha}}^z)$ is the postulated model for Y^z where $\hat{\boldsymbol{\alpha}}$ is the estimator of the coefficient for $z = 0, 1$, $e(\cdot, \hat{\boldsymbol{\beta}})$ is the postulated model for the propensity score where $\hat{\boldsymbol{\beta}}$ is the estimator of the coefficient. For each estimator, the condition for consistency has two parts: the covariate set satisfy the strong ignorability assumption, and the functional forms are correct.

Estimator	Strong Ignorability	Functional Forms
Regression	$\boldsymbol{A}$ for outcome	$m^z(\boldsymbol{A}, \hat{\boldsymbol{\alpha}}^z) \xrightarrow{p} \mathbb{E}(Y^z \mid \boldsymbol{A})$
IPW	$\boldsymbol{B}$ for selection	$e(\boldsymbol{B}, \hat{\boldsymbol{\beta}}) \xrightarrow{p} \mathbb{E}(Z \mid \boldsymbol{B})$
DR	$\boldsymbol{X} = \boldsymbol{A} \cup \boldsymbol{B}$	$m^z(\boldsymbol{A}, \hat{\boldsymbol{\alpha}}^z) \xrightarrow{p} \mathbb{E}(Y^z \mid \boldsymbol{A}) = \mathbb{E}(Y^z \mid \boldsymbol{X})$ or $e(\boldsymbol{B}, \hat{\boldsymbol{\beta}}) \xrightarrow{p} \mathbb{E}(Z \mid \boldsymbol{B}) = \mathbb{E}(Z \mid \boldsymbol{X})$

Table 2: Biases of estimates with 95% simulation confidence intervals when both models are correctly specified. In the first three cases all the estimators are consistent, but in the last case the two DR estimators are inconsistent even though the corresponding outcome model and the selection model are both correct on their own.

Covariates for Y	$\{W, V\}$	$\{W, V, M\}$	$\{W, U\}$	$\{W, U, M\}$
Regression	**0.000** $(-0.004, 0.005)$	**0.000** $(-0.005, 0.004)$	**0.000** $(-0.005, 0.005)$	**0.000** $(-0.004, 0.005)$
Covariates for Z	$\{W\}$	$\{W, U\}$	$\{W, V\}$	$\{W, V, M\}$
IPW	**0.000** $(-0.004, 0.005)$	**0.000** $(-0.005, 0.005)$	**0.000** $(-0.004, 0.005)$	**0.000** $(-0.004, 0.005)$
All Covariates	$\{W, V\}$	$\{W, U, V, M\}$	$\{W, U, V\}$	$\{W, U, V, M\}$
AIPW	**0.000** $(-0.004, 0.005)$	**0.000** $(-0.004, 0.005)$	**0.000** $(-0.004, 0.005)$	0.052 $(0.048, 0.057)$
IPWR	**0.000** $(-0.004, 0.005)$	**0.000** $(-0.004, 0.005)$	**0.000** $(-0.005, 0.004)$	0.053 $(0.048, 0.057)$

* Prima facie estimator: 0.299 $(0.294, 0.304)$

Table 3: Biases of estimates with 95% simulation confidence intervals when one model is incorrect due to colliders. In all the four cases one model is correctly specified while the other model leads to inconsistent estimators due to collider bias, and the two DR estimators are inconsistent.

Covariates for Y	$\{M\}$	$\{W, M\}$	$\{W, U\}$	$\{W\}$
Regression	0.172 (0.167, 0.177)	−0.144 (−0.149, −0.140)	**0.000** (−0.005, 0.005)	**0.000** (−0.004, 0.005)
Covariates for X	$\{W\}$	$\{W, V\}$	$\{M\}$	$\{W, M\}$
IPW	**0.000** (−0.004, 0.005)	**0.000** (−0.004, 0.005)	0.175 (0.170, 0.179)	−0.142 (−0.146, −0.137)
All Covariates	$\{W, M\}$	$\{W, V, M\}$	$\{W, U, M\}$	$\{W, M\}$
AIPW	−0.134 (−0.138, −0.130)	−0.144 (−0.149, −0.140)	−0.132 (−0.137, −0.128)	−0.142 (−0.146, −0.137)
IPWR	−0.134 (−0.138, −0.129)	−0.145 (−0.149, −0.140)	−0.135 (−0.139, −0.130)	−0.142 (−0.146, −0.137)

* Prima facie estimator: 0.299 (0.294, 0.304)

3 Figures

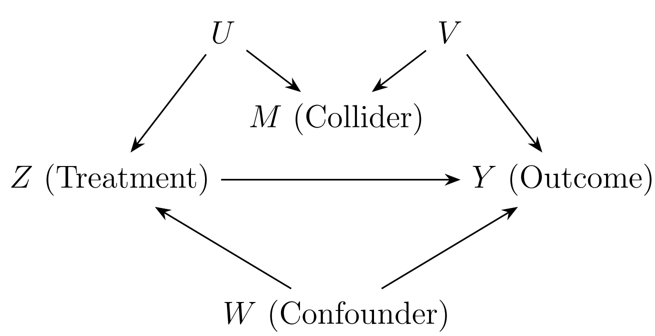

Figure 1: Causal relationship among variables. There are three paths between Z and Y: $Z \to Y$, $Z \leftarrow W \to Y$ and $Z \leftarrow U \to M \leftarrow V \to Y$. The first is the only causal path and the effect to be estimated. The second path results in confounding bias because of the confounder W. The third path is naturally blocked because of the collider M, but will be open if M is conditioned on.

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