How do Balance Scales Shape K-2 Students' Understandings of Equations?

The concept of mathematical equivalence is foundational to arithmetic and algebraic thinking (Baroody & Ginsburg, 1983; Carpenter, Franke, & Levi, 2003; Kieran, 1981). Unfortunately, decades of research have demonstrated that elementary and middle school students consistently struggle to develop a robust understanding of the equal sign as a relational symbol indicating an equivalence relationship (Behr, Erlwanger, & Nichols, 1980; Matthews, Rittle-Johnson, McEldoon, & Taylor, 2012; McNeil & Alibali, 2005).

One model often used to teach elementary and middle grades students about mathematical equivalence is a balance model (e.g., Alibali, 1999; Fyfe, McNeil, & Borjas, 2015; Linchevski & Herscovics, 1996). Rationales for the use of such a model include the analogy that can be made between balancing a scale and balancing an equation (e.g., Warren & Cooper, 2005), the model's grounding in physical experience (e.g., Alibali, 1999; Araya et al., 2010) and the link such a model provides between concrete experience and abstract representations (e.g., Fyfe et al., 2015). In a review of 34 research articles in which the use of a balance model was reported, Otten, Van den Heuvel-Panhuizen, and Velduis (2019) found great diversity in terms of the specific types of models used, the purposes of their use, and learning outcomes for students. While their analysis identified some trends, the authors concluded that more research is necessary on the effects of using balance models of different types with students of varying levels of algebra experience.

The research that will be presented focuses on students' thinking about mathematical equivalence and equations in the context of their work with two kinds of balance scales—a pan balance (a scale with containers to hold objects on each side) and a number balance (a t-shaped scale with numbers 1–10 on each side where individual pegs can be hung; see Figure 1).

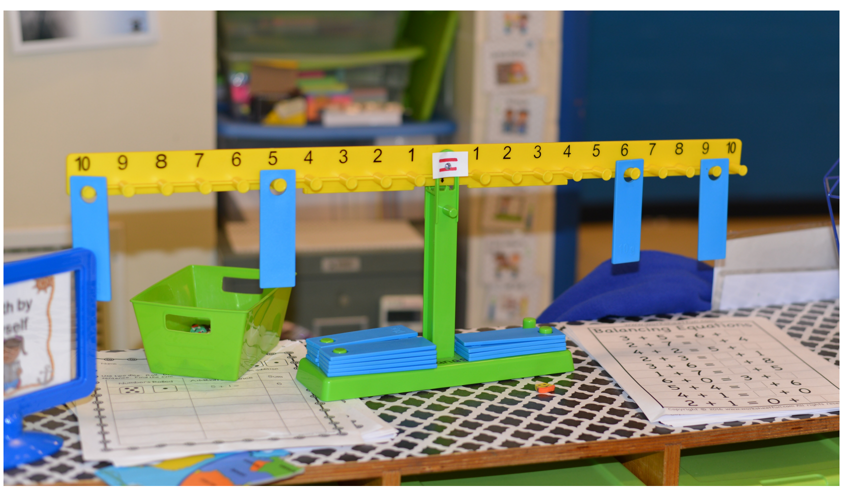

Figure 1. Number balance representing the equation 5 + 10 = 6 + 9.

Our goal was to identify how students viewed the equal sign and reasoned with equations through their work with these tools. We also made note of any misconceptions students demonstrated about equations or the equal sign that these tools helped reveal. Our research question was the following: *How does students' use of balance scales mediate their relational understanding of the equal sign?*

Method

The data we report in this paper come from a larger cross-sectional study in which we built on previous work (see Blanton et al. [2015] and Fonger et al. [2018] for a details regarding the development of the curricular framework used in previous and current studies) to develop, test, and refine an instructional intervention designed to engage Grades K–2 students in early algebraic thinking and studied this thinking in the context of the intervention. We focus here on data collected from students who participated in task-based interviews concerning the meaning of the equal sign. While the data we will share comes from interviews conducted before, during, and after an early algebra intervention in which balance scales were used as part of instruction, our focus here is not on pre-to-post growth in students' understandings. Rather, we focus on observations made across the interviews that shed light on how the balance models mediated students' relational thinking.

Participants

Interview participants included 21 students (ten kindergarten students, six Grade 1 students, and five Grade 2 students) from two schools who were posed tasks that addressed the meaning of the equal sign and work with equations. We selected one of the schools based on its diversity in terms of student ethnicity (64% students of color), socioeconomic status (63% of students qualifying for free or reduced-price lunch), and language status (27% English Language Learners). The second school was selected based on the fact that it serves a high percentage of students with learning difficulties, including learning differences and disabilities.

Data collection

Students participated in teaching experiment interviews in the spirit of Vygotsky's (1978) laboratory experiments that "provide maximum opportunity for the subject to engage in a variety of activities that can be observed, not just rigidly controlled" (p. 12) and furnish data that is "not performance level as such but the methods by which the performance is achieved" (p. 13). These interviews addressed a wide range of early algebra content and took place prior to the early algebra intervention, mid-way through the intervention, and after the conclusion of the intervention. The particular tasks that are the focus of this paper are those that engaged students in solving true-false and open number sentences (see Table 1 for specific tasks included in the interview protocol). Students were often encouraged to show their thinking using balance scales, while at other times the scales were simply available for students' use at their discretion.

Table 1

Equation tasks posed to students

Kindergarten		**Grades 1–2**	
True/False equations	5 = 5	*True/False equations*	7 = 3 + 2
	7 = 3 + 2		8 = 8
	4 = 4		3 + 5 = 7
	5 = 1 + 4		9 = 4 + 3

4 + 2 = 3		4 + 5 = 9 + 3
2 + 3 = 5 + 1		3 + 6 = 2 + 7
3 = 1 + 2		
4 = 1 + 2	*Open number sentences*	4 = 3 + ___
2 + 4 = 3 + 3		6 = ___
6 = 3 + 2		3 + ___ = 5
4 = 2 + 2		3 + 2 = ____ +3
		4 + 1 = 1 + ___
		4 + ___ = 8 + 4

Data analysis

Given that the goals of the project include the identification of instructional strategies to help a diverse population of learners be successful with early algebra, our analysis of the videotaped interviews started with a grounded approach in which coders freely noted what they noticed in students' responses about mathematical equivalence and the instructional prompts, discussion, and tools that may have led to such responses. At least two coders watched each video and discussed observations with each other. After this first pass, discussion among all coders focused on the identification of particular categories that would provide the focus for subsequent analyses. These categories were first posited from the pre-intervention interview data and then extended and refined with the inclusion of the subsequent data. Subsequent analyses involved a primary coder watching the interview video and populating the categories with interview excerpts while a second coder watched the video and noted agreement or disagreement with the first coder. Any disagreements were discussed until resolved. The majority of categories that emerged from the data—and what we focus on here—concerned the affordances of the balance scales for mediating students' relational understandings of the equal sign as well as the mathematical difficulties and misconceptions these tools helped reveal.

Results

Our analysis revealed several (related) categories of affordances of the balance scales as well as a difficulty revealed by these tools. Specifically, we found that the balance scales helped challenge students' initial interpretations equations, and, more specifically, encouraged trial-and-error equation solving, encouraged productive "tinkering" and play, and helped students notice equation structure. In addition, the scales helped reveal students' difficulty viewing an equation as a comparison of two equivalent quantities, a difficulty we believe is consistent with an operational conception of the equal sign. In the poster presentation, we will elaborate on each of these categories and provide illustrative examples from our interviews.

The "take home" findings from this research suggest that students' work with the balance scales in the context of mathematical equivalence tasks helped them develop the understandings that a) the equal sign indicates the equivalence of two quantities, b) equations can take various forms, c) equations do not need to have operations, and d) equations can be reasoned about without computation. We will elaborate on these findings by sharing our observations around how the interaction of tools and tasks shaped students' thinking.

Conclusion

An important goal in the shift towards introducing core algebraic ideas in the elementary grades is the engagement of *all* learners in algebraic thinking. Meeting this goal requires the use of instructional strategies, tasks, and mathematical tools that build on what students already know and help move their thinking forward. The Kindergarten through second-grade students whom we interviewed showed that they were able to move forward in their understanding of equations and the meaning of the equal sign through the use of balance scales.

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