# Interpretation and Effect Size Measures for Random Effects in Multilevel Survival Models

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## Multilevel Survival Models

Discrete multilevel survival model:

$$\text{logit}(p_{ij}) = \gamma_1 * t1_{ij} + \gamma_2 * t2_{ij} + \gamma_3 * t3_{ij} + \gamma_4 * t4_{ij} + \gamma_5 * t5_{ij} + u_{0j}$$

$$\text{where } logit(p) = ln(\frac{p}{1-p})$$

Continuous multilevel Cox regression (frailty) model:

$$h_i(t) = h_0(t)\exp(u_j)$$

Where p represents the probability of the event occurring, t1-t5 represent five discrete time points, γ represent slope coefficients, u represents the random effect for group, i represents level-1 individuals, j represents level-2 groups, and h0(t) represents the baseline hazard function.

### Effect Size Measures

- Intraclass correlation coefficient (ICC) = $\sigma^2/(\sigma^2 + \pi^2/3)$ where $\sigma^2$ is the variance of the random effect (Rodriguez & Elo, 2003)
- Median odds ratio (MOR) = $\exp(\sqrt{2\sigma^2}\phi^{-1}(.75))$ where $\phi^{-1}$ indicates the inverse of the standard normal cumulative distribution function (Austin et al., 2017)
- Median hazard ratio (MHR) = upper quantile of $F(2\sigma^{-2}, 2\sigma^{-2})$ distribution (Austin et al., 2017)

## Research Question

How does the choice of a discrete versus continuous multilevel survival model relate to the information about random effect size provided for various datasets?

## Methods

Monte Carlo simulation:

- 500 simulations/condition
- 2*2*3=12 conditions total
- Average probability of censoring = 0.3
- Number of time periods = 5

Variables that vary by condition:

- Level 1 sample size: 5 and 30
- Level 2 sample size: 10 and 40
- Nesting: small, medium, and large effect

Models:

- Discrete survival model with multilevel logistic regression
- Continuous multilevel Cox model with Gamma random effects

## Results

**Table 1**

Average value of ICC, MOR, & MHR for each of 12 conditions.

Each value based on 500 replications. SE represents the standard deviation of the given measure. NL1 = group size; NL2 = number of groups; L2SD = levels of nesting.

| NL1 | NL2 | L2SD | ICC | SE | MOR | SE | MHR | SE |
|---|---|---|---|---|---|---|---|---|
| 5 | 10 | 0.2 | 0.04 | 0.05 | 1.33 | 0.38 | 1.03 | 0.07 |
| 30 | 10 | 0.2 | 0.04 | 0.03 | 1.39 | 0.22 | 1.09 | 0.07 |
| 5 | 40 | 0.2 | 0.04 | 0.03 | 1.39 | 0.24 | 1.01 | 0.04 |
| 30 | 40 | 0.2 | 0.04 | 0.02 | 1.45 | 0.1 | 1.13 | 0.05 |
| 5 | 10 | 0.3 | 0.08 | 0.08 | 1.61 | 0.59 | 1.1 | 0.16 |
| 30 | 10 | 0.3 | 0.08 | 0.07 | 1.68 | 0.46 | 1.14 | 0.08 |
| 5 | 40 | 0.3 | 0.08 | 0.05 | 1.67 | 0.31 | 1.08 | 0.11 |
| 30 | 40 | 0.3 | 0.09 | 0.03 | 1.74 | 0.2 | 1.05 | 0.08 |
| 5 | 10 | 0.4 | 0.12 | 0.11 | 1.93 | 0.8 | 1.19 | 0.22 |
| 30 | 10 | 0.4 | 0.14 | 0.11 | 2.12 | 1.03 | 1.16 | 0.09 |
| 5 | 40 | 0.4 | 0.14 | 0.06 | 2 | 0.36 | 1.23 | 0.16 |
| 30 | 40 | 0.4 | 0.16 | 0.06 | 2.13 | 0.36 | 1.01 | 0.04 |

**Table 2**

Correlations between conditions and effect size values.

NL1 = group size; NL2 = number of groups; L2SD = levels of nesting. Each value based on 12 conditions * 500 replications = 6000 total replications.

| | NL1 | NL2 | L2SD | ICC | MOR | MHR |
|---|---|---|---|---|---|---|
| ICC | 0.06 | 0.05 | 0.53 | | | |
| MOR | 0.08 | 0.05 | 0.47 | 0.97 | | |
| MHR | -0.04 | -0.13 | 0.26 | 0.49 | 0.44 | |

## Conclusions

Based on the present results, the degree of nesting may be under-represented with the MHR based on the Cox regression model compared with the MOR based on the logistic regression model. With the present conditions, particularly with only 5 time points, the estimates based on the continuous Cox model appear to be somewhat unstable.

References

Austin, P. C., Wagner, P., & Merlo, J. (2017). The median hazard ratio: A useful measure of variance and general contextual effects in multilevel survival analysis. *Statistics in Medicine, 36*, 928-938. DOI: 10.1002/sim.7188.

Rodriguez, G. & Elo, I. (2003). Intra-class correlation in random-effects models for binary data. *The Stata Journal, 3*(1), 32-46.

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