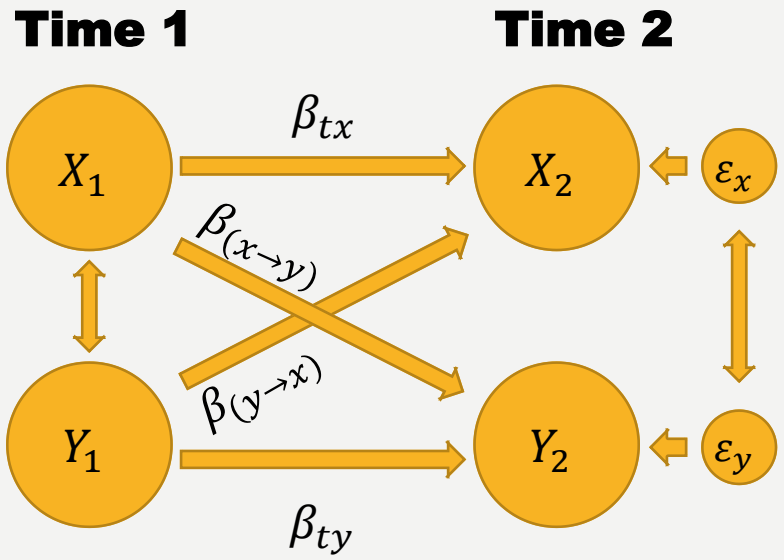
Examining the Causal Direction of Synchronous Effects in Structural Panel Models

Using Residual-predictor Independence Tests to Identify Model Mis-specifications

Xintong Li

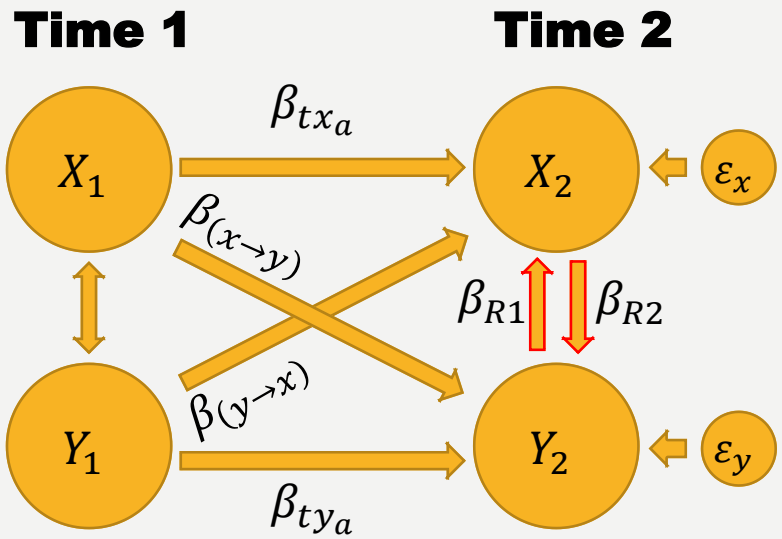
University of Missouri

Cross-lagged Panel Model (CLPM)



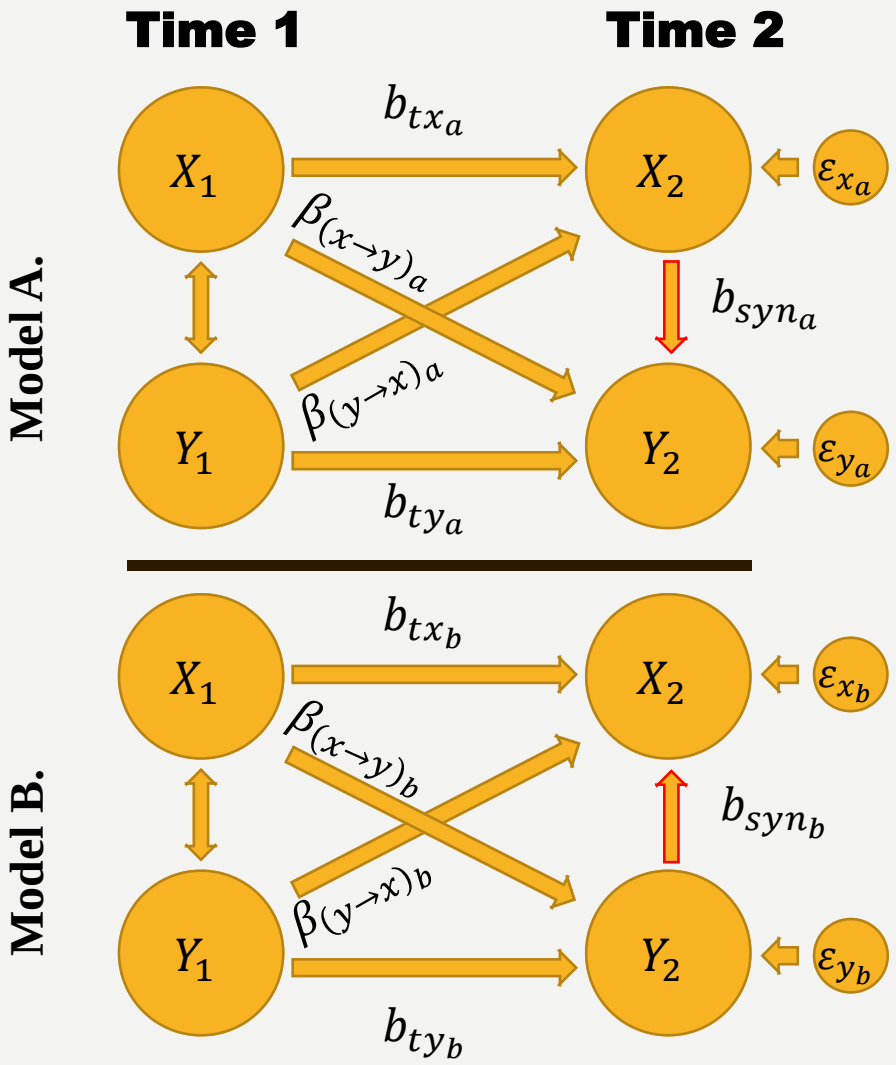
- Cross-lagged panel models are structural equation models that estimate the directional influence/causal predominance/ causal preponderance over time between two focal variables, which are collected at multiple time points (Finkel, 1995, Newsom, 2015).
- Major Assumptions:
  - No measurement error
  - Appropriate time interval for the lagged effect
  - No confounder
  - **Zero synchronous effect**

CLPM with Synchronous Effect (1)



- Reciprocal Synchronous Effect Model
  - Non-recursive
  - Not Identifiable without instrumental variable(s)

CLPM with Synchronous Effect (2)

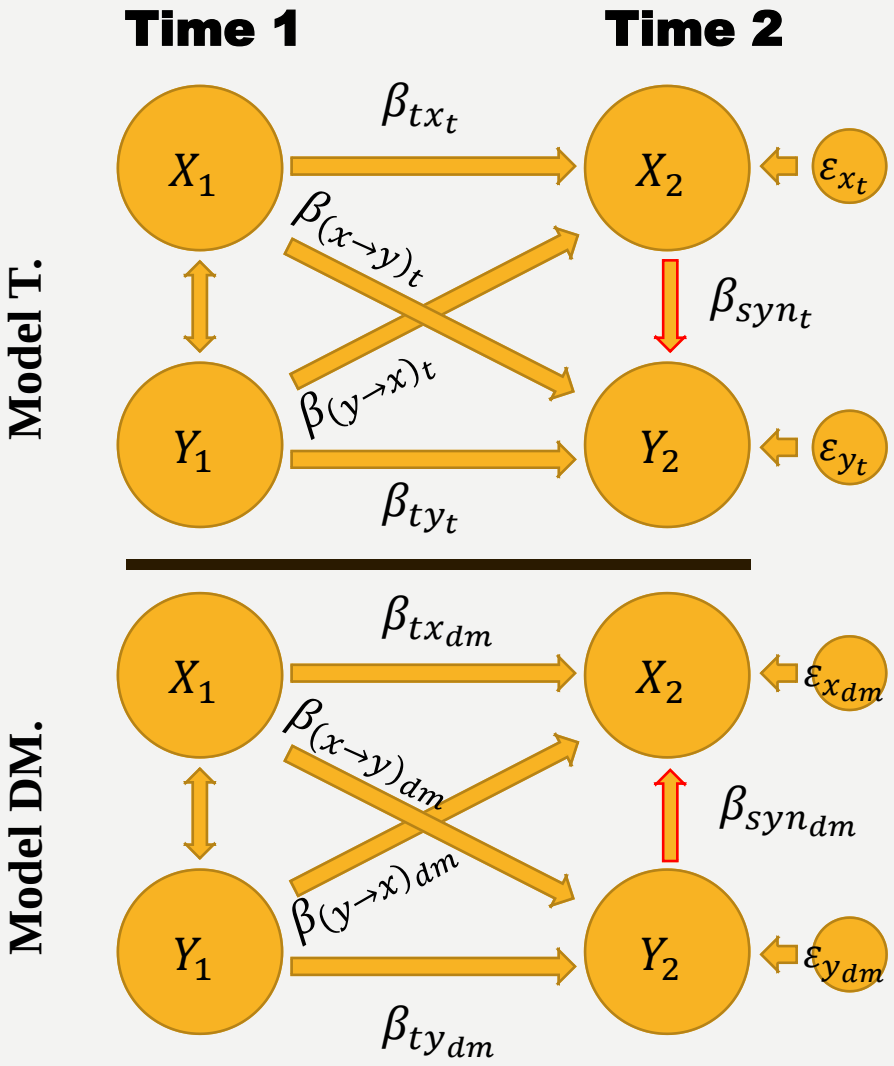


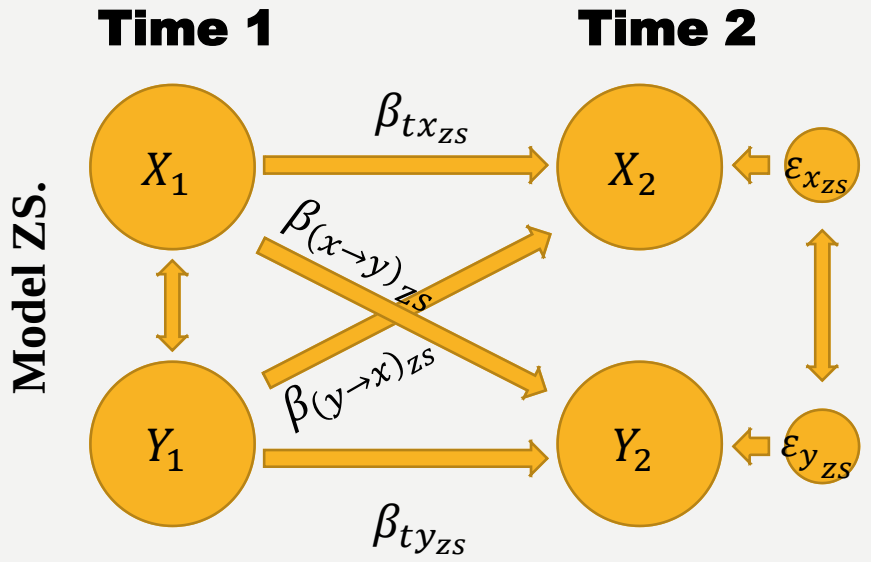
- Competing Synchronous Effect Model
  - **[What]** Model Selection → Parameter Estimation
  - **[One option]** Theory based model selection
  - **[Why]** Mis-specified model → biased parameter estimates?
  - **[Challenge]** The same model fit
    - two waves
    - multiple waves
  - **[Solution]** Residual-predictor independence tests

Research Question 1

Will a mis-specified direction of synchronous effect or the neglecting of the synchronous effect lead to erroneous causal conclusions in cross-lagged panel models?

Monte Carlo Simulation: Models

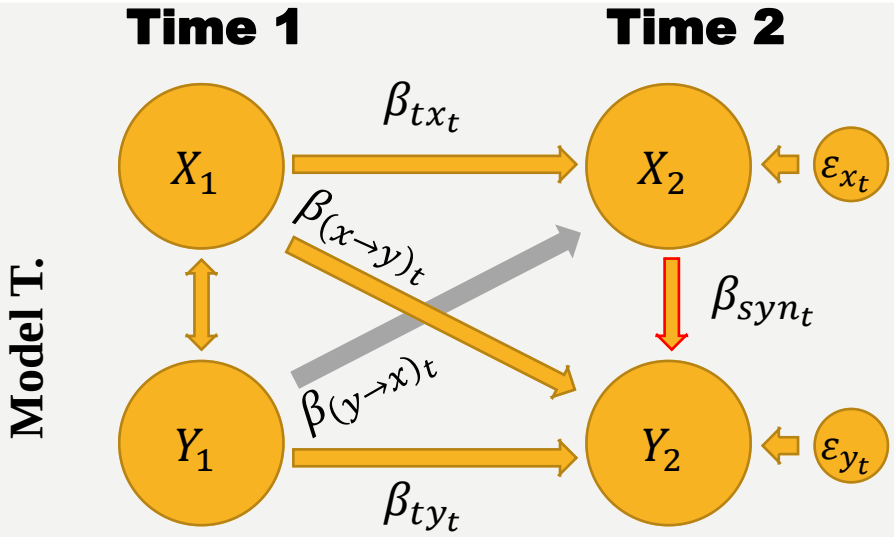




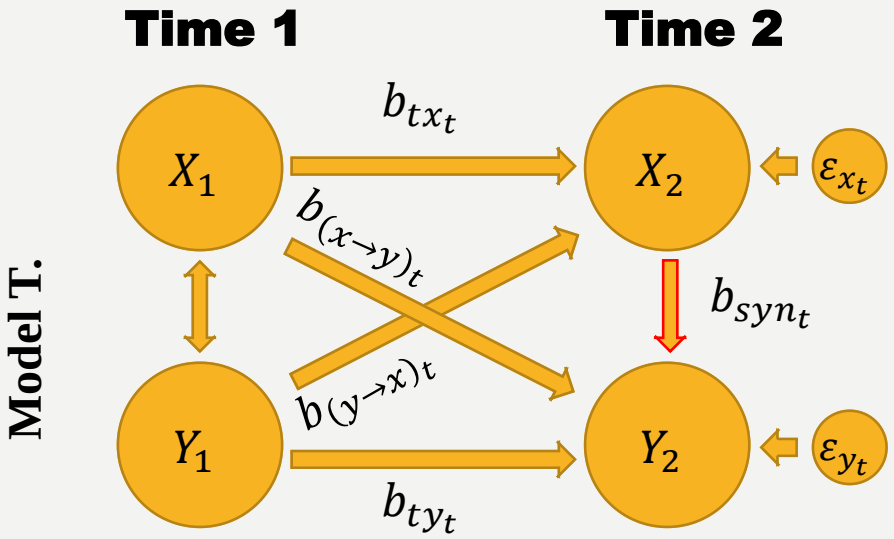
Monte Carlo Simulation: Parameters

Simulations (2 × 2 × 2=8)	
Synchronous effect	b_{syn_t} ∈**{0.14, 0.39}**
Cross-lagged effects	$\beta_{(x\rightarrow y)_t}$ ∈**{0, .39}** ; $\beta_{(y\rightarrow x)_t}$ ∈ { 0 }
Autoregressive effects	β_{tx_t} ∈ { 0.59 }; β_{ty_t} ∈ { 0.59 }
Shape of *x* and *y* at Time 1	γ_{x_1} ∈ { **1, 1. 4** }; γ_{y_1} ∈ { 1.4 }; Gamma Distribution
Shape of residual terms	γ_{ε_x} ∈ { 1.4 }; γ_{ε_y} ∈ { 1.4 }; Gamma Distribution
Correlation of Time 1 *x* and *y*	$\rho_{x_1y_1}$ ∈ {0.39}

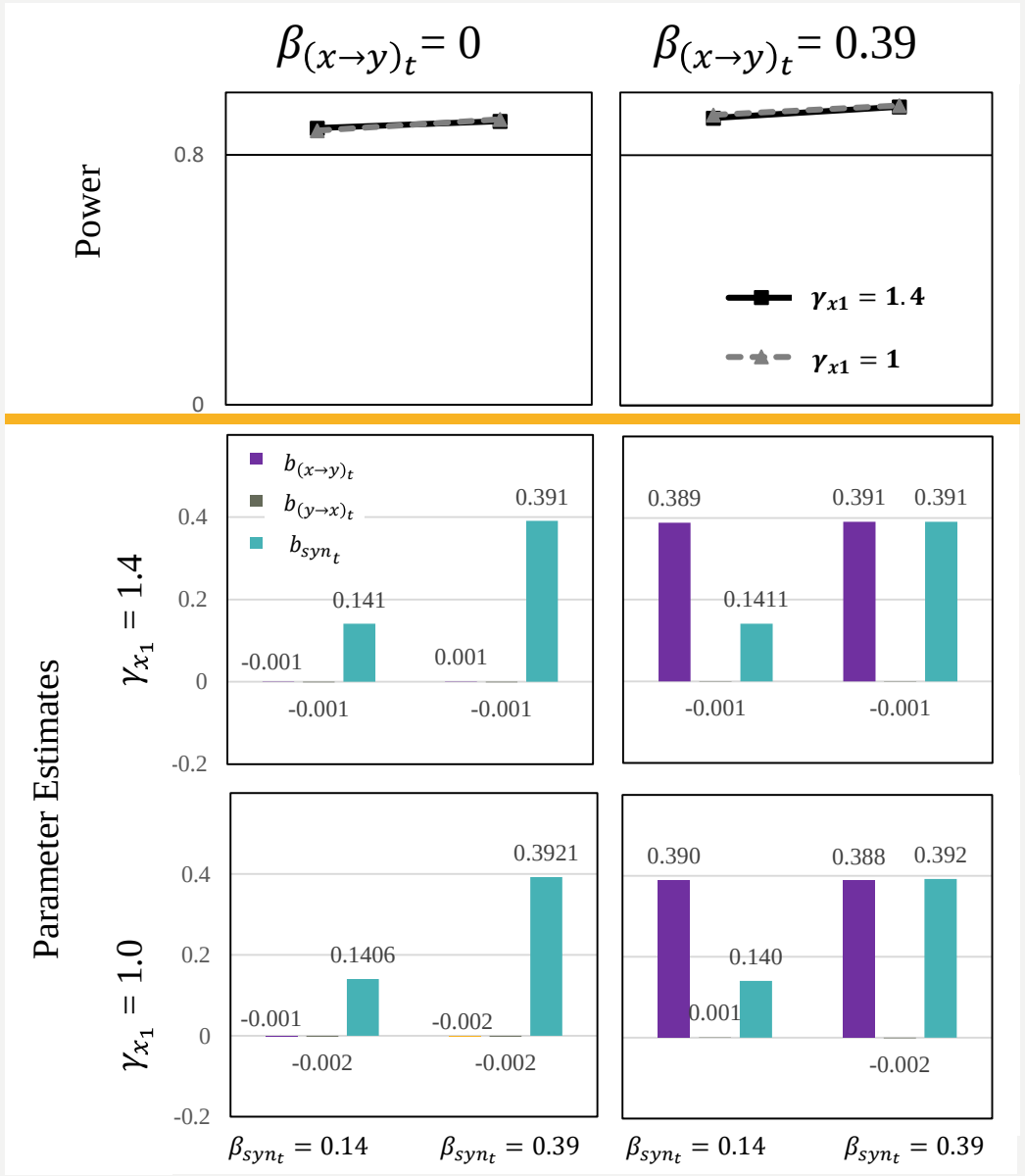
- N = 800 (Wiedermann, Li, & von Eye, 2019).
- Iteration =500



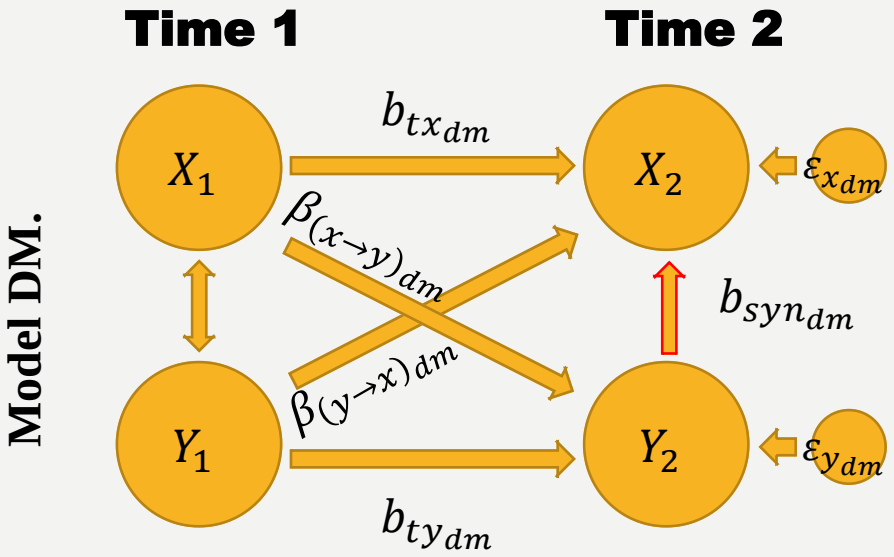
Results: True Model



Model Selection Criterion	
No cross-lagged effect $\beta_{(x\to y)_t} = 0$	Non-zero cross-lagged effect $\beta_{(x\to y)_t} = 0.39$
$p_{\beta_{(x\to y)}} \geq .05$ and $p_{\beta_{(y\to x)}} \geq .05$ and $p_{syn_t} < .05$	$p_{\beta_{(x\to y)}} < .05$ and $p_{\beta_{(y\to x)}} \geq .05$ and $p_{syn_t} < .05$

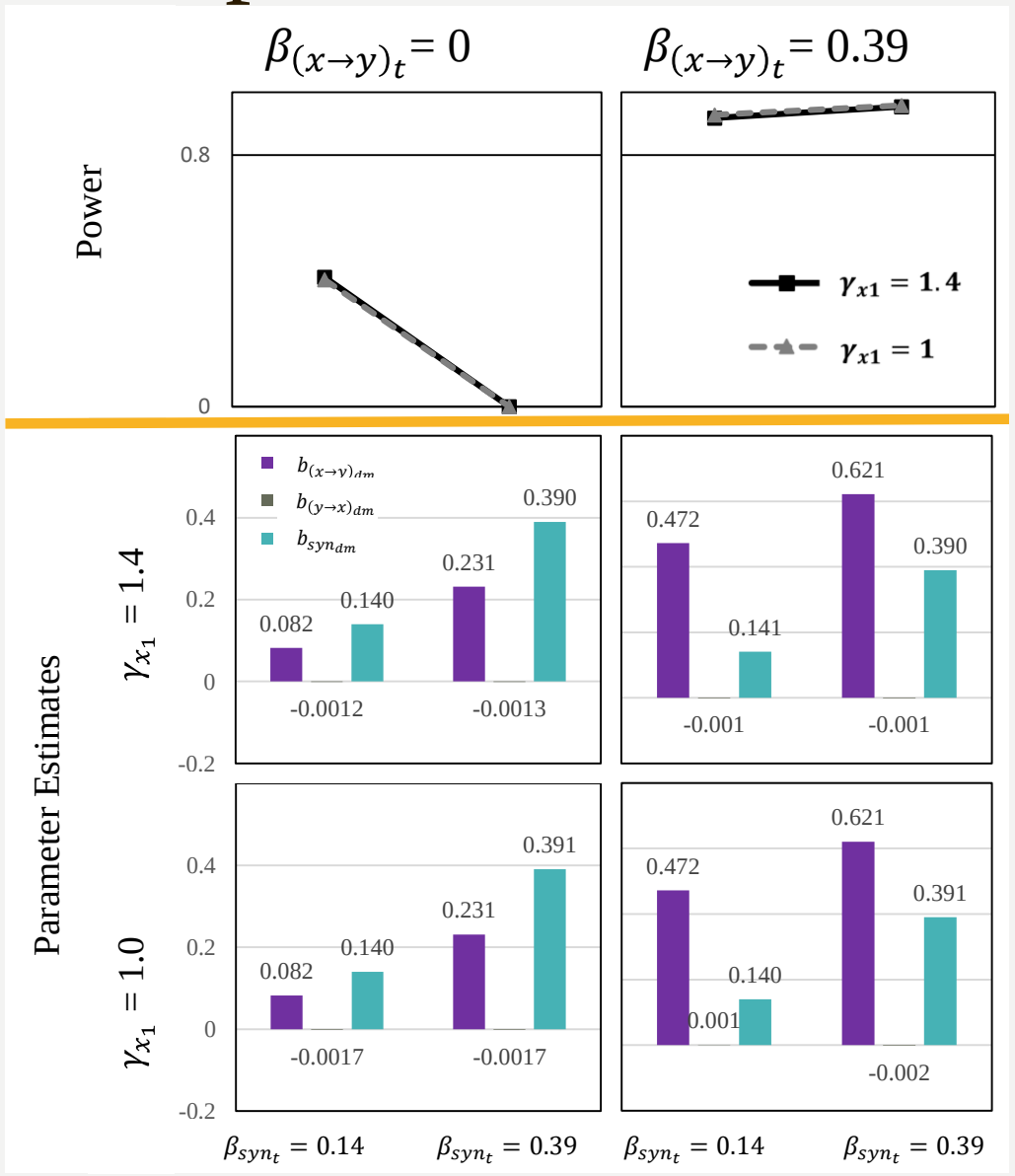


Results: Directionally Mis-specified Model

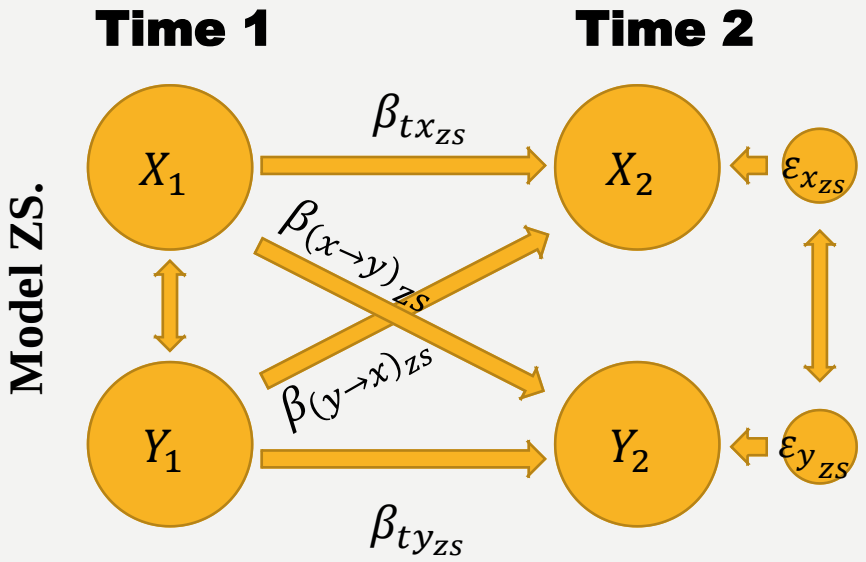


Model Selection Criterion	
No cross-lagged effect $\beta_{(x\to y)_t} = 0$	Non-zero cross-lagged effect $\beta_{(x\to y)_t} = 0.39$
$p_{\beta_{(x\to y)}} \geq .05$ and $p_{\beta_{(y\to x)}} \geq .05$ and $p_{syn_t} < .05$	$p_{\beta_{(x\to y)}} < .05$ and $p_{\beta_{(y\to x)}} \geq .05$ and $p_{syn_t} < .05$

Note that the empirical power is based on the correct causal inference using cross-lagged effects only.

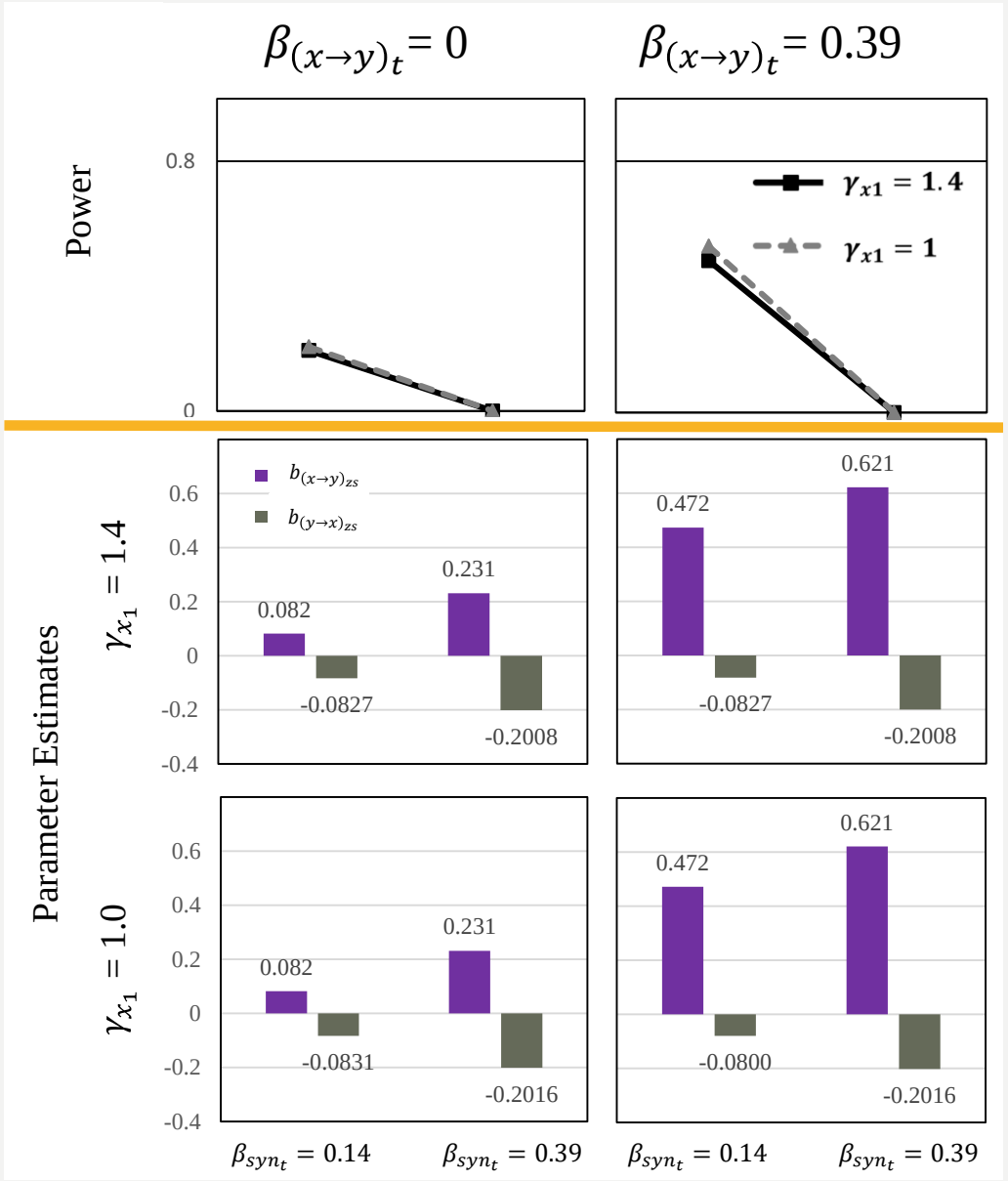


Results: Zero Synchronous Effect Model

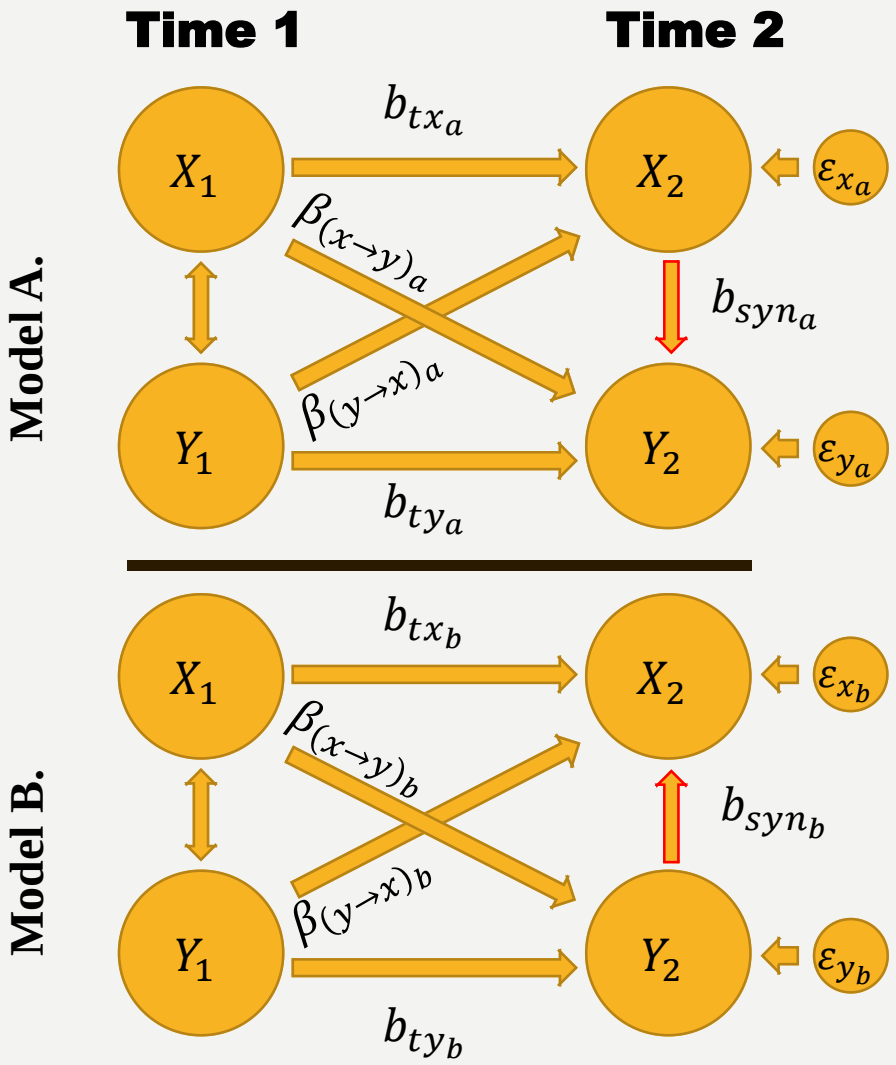


Model Selection Criterion	
No cross-lagged effect $\beta_{(x\to y)_t} = 0$	Non-zero cross-lagged effect $\beta_{(x\to y)_t} = 0.39$
$p_{\beta_{(x\to y)}} \geq .05$ and $p_{\beta_{(y\to x)}} \geq .05$	$p_{\beta_{(x\to y)}} < .05$ and $p_{\beta_{(y\to x)}} \geq .05$

Note that the empirical power is based on the correct causal inference using cross-lagged effects only.

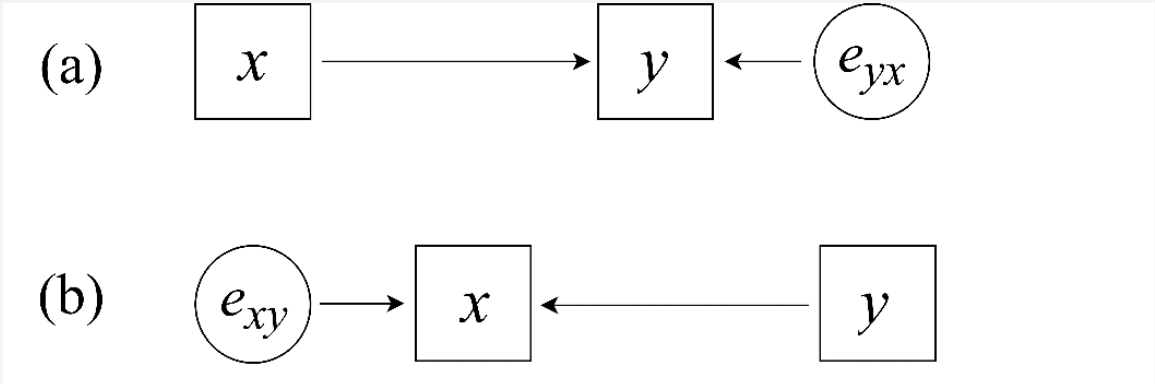


CLPM with Synchronous Effect (2)



- Competing Synchronous Effect Model
  - **[What]** Model Selection → Parameter Estimation
  - **[One option]** Theory based model selection
  - **[Why]** Mis-specified model → biased parameter estimates.
  - **[Challenge]** The same model fit
    - two waves
    - multiple waves
  - **[Solution]** Residual-predictor independence tests

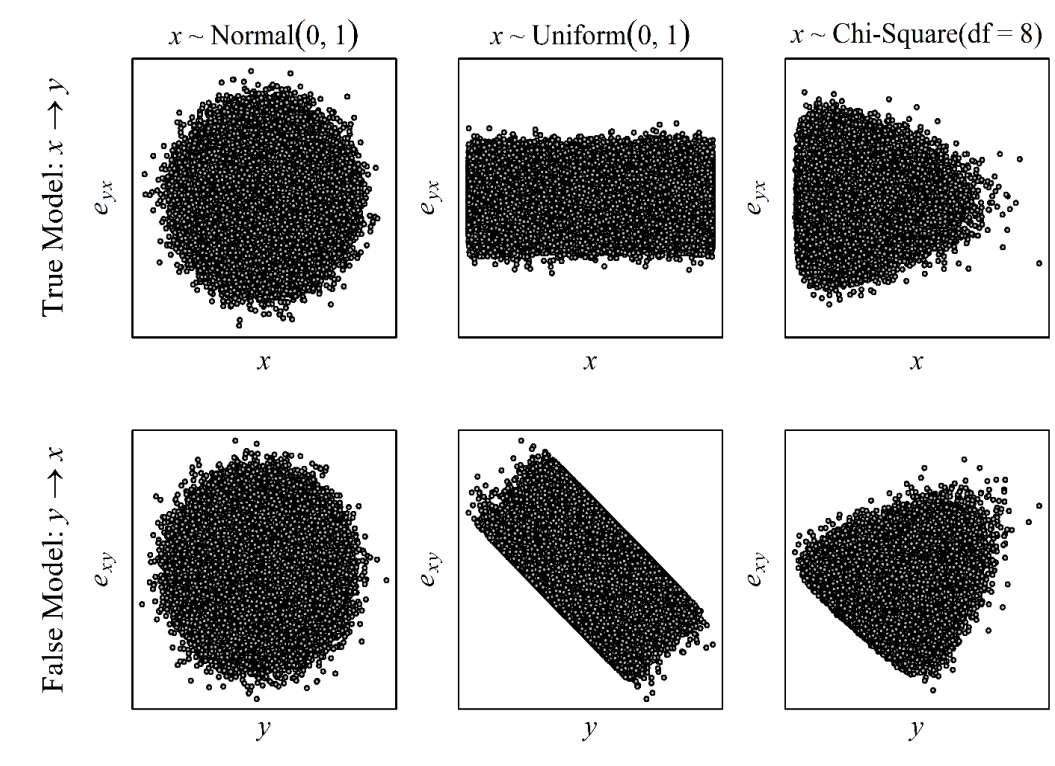
Model Selection with Observational Data (1)



	True Model (a) $\boldsymbol{y = \alpha_a + \beta_a x + \varepsilon_a}$	**Mis-specified Model (b)** $\boldsymbol{x = \alpha_b + \beta_b y + \varepsilon_b}$
Predictor-residual Independence	The true predictor (x) and the true error term (ε_y) are independent $x \perp \varepsilon_y$	The false predictor (y) and the false error term (ε_x) are non-independent when the "true" predictor x is non-normal. (Shimizu et al., 2011). $y \not\perp \varepsilon_x$

- Li, X. & Wiedermann, W. (2019). Conditional Direction Dependence Analysis: Evaluating the Causal Direction of Effects in Linear Models with Interaction Terms. *Multivariate Behavioral Research, (in press)*
- Wiedermann, W., & Sebastian, J. (2019). Direction dependence analysis in the presence of confounders: Applications to linear mediation models using observational data. *Multivariate behavioral research*, 1-21.
- Wiedermann, W., Li, X., & von Eye A. (2019). Testing the causal direction of mediation effects in randomized intervention studies. *Prevention Science, 20, 419-430*.
- Wiedermann, W., & Li, X. (2018). Direction dependence analysis: Testing the direction of effects in linear models with implementation in SPSS. *Behavior Research Methods*, 50 (4), 1581-1601.

Predictor-residual Independence



Bivariate patterns of predictors and residuals of competing linear models ($x \rightarrow y$ and $y \rightarrow x$) for normal, and non-normal "true" predictors.
(Wiedermann & Li, 2018)

- In the normal case, the two models cannot be distinguished from each other.
- In the non-normal scenarios, the two competing models *are mutually distinguishable*. Clear dependence structures occur in the mis-specified model.
- The normal distribution constitutes the special case where competing models cannot be uniquely distinguished because uncorrelatedness implies stochastic independence in the normal domain (cf. Hoyer et al., 2008).

Predictor-residual Independence Tests

- Model selection can be based on any independence test that is able to pick up dependence structures beyond *first order correlations*, as OLS residuals and predictors are always uncorrelated, or linearly independent.

- Stochastic independence ≠ linear independence

Homoscedasticity Tests

- Beyond the conventional explanations of heteroscedasticity, another type of source of non-constant variance is from directionally mis-specified models when the “true” predictor are non-normal (Wiedermann et al., 2017).

- Breusch–Pagan test (Breusch & Pagan, 1979) and the robust version of Breusch–Pagan test (Koenker, 1981; Koenker & Bassett Jr, 1982)

Omnibus independence tests

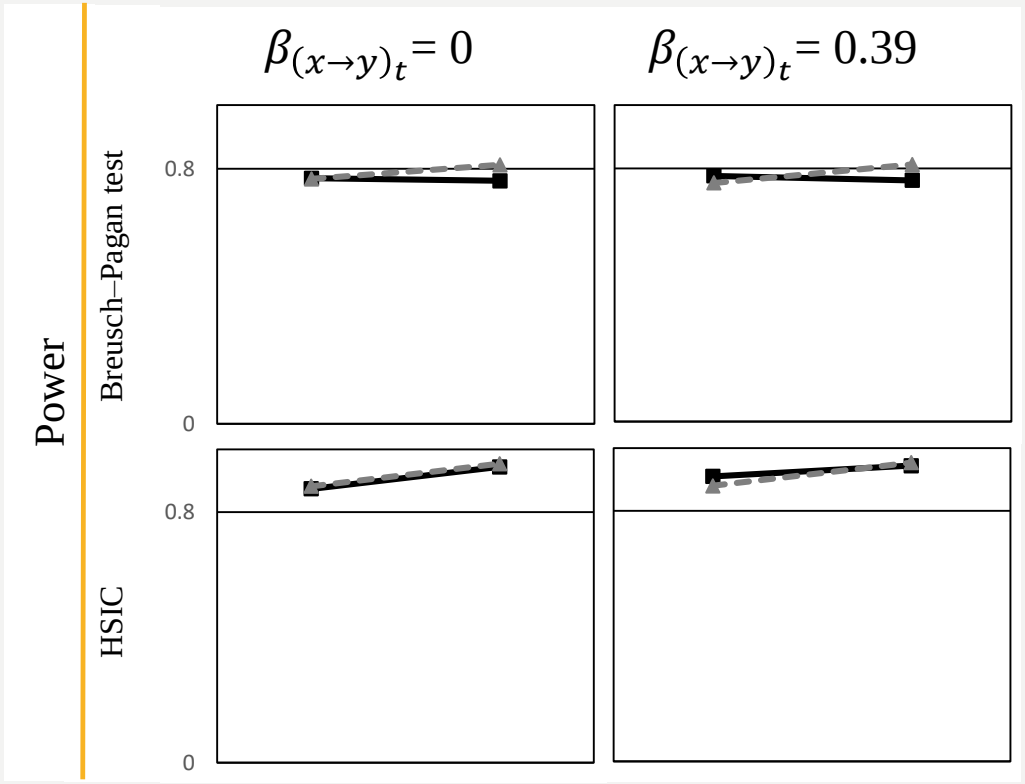
- Hilbert-Schmidt Independence Criterion (HSIC; cf. Gretton et al., 2008)
- The HSIC evaluates the independence of functions of random variables on the basis of Euclidian distance matrices and is provably omnibus in detecting any dependence between two random variables as $n \to \infty$.
- The HSIC is also introduced in the context of testing the independence of predictors and error terms of linear regression models with a bootstrap approach to approximate the null distribution of the test statistic (Sen and Sen, 2014).

Research Question 2

Can Homoscedasticity tests and Hilbert Schmidt Independence Criterion (HSIC) inform the selection of structural panel models with competing direction of a synchronous effect?

Simulation Results: Independence Tests

Model Selection Criterion	
No cross-lagged effect $\beta_{(x\rightarrow y)_t} = 0$	Non-zero cross-lagged effect $\beta_{(x\rightarrow y)_t} = 0.39$
$p_{BP_{(x\rightarrow y)}} \geq .05$ and $p_{BP_{(y\rightarrow x)}} < .05$	
$p_{HSIC_{(x\rightarrow y)}} \geq .05$ and $p_{HSIC_{(y\rightarrow x)}} < .05$	



For more details see e.g.,

Wiedermann, W., & Sebastian, J. (2019). **Sensitivity analysis and extensions of testing the causal direction of dependence: A rejoinder to Thoemmes (2019)**. *Multivariate Behavioral Research*, (*in press*).

Wiedermann, W., & Li, X. (2018). **Direction dependence analysis: Testing the direction of effects in linear models with implementation in SPSS**. *Behavior Research Methods*, 50 (4), 1581-1601.

Wiedermann, W., Artner, R., & von Eye, A. (2017). **Heteroscedasticity as a basis for direction dependence in reversible linear regression models.** *Multivariate Behavioral Research, 52, 222-241.*

Research Question 3

Can the proposed tests be used with empirical data to identify the direction of a synchronous effect?

An Empirical example:

Teacher Student Relation and Teachers' Cognitive Engagement Practices.

An Empirical Example from NEE

- Network for Educator Effectiveness (NEE; neeadvantage.com)
  - An authentic comprehensive educator evaluation system that provides services to over 283 school districts throughout Missouri
  - Teacher Effectiveness Student Survey (TESS) is a modular survey and includes 39 indicators of teaching practices that school districts can choose from according to their priority.
    - *Cognitive engagement* (CE) is one of the most prioritized teaching practices, which measure the degree to which a teacher cognitively engages students in the content in their teaching practices. (e.g., "The teacher wants us to ask questions during lessons.") 5 statement items are included
    - *Teacher Student Relation* (TSR) Three items address whether a teacher creates positive relationships with students (e.g., This teacher knows me and cares about me). 4 statement items are included
    - 4 point scale: ("Not true", "Sort of true", "True", "Very true")

An Empirical Example: Participants

- 613 Teachers in districts that have both *cognitive engagement* and *teacher student relation* prioritized across school year 2017-2018 and 2018-2019.

- Teachers were rated by students above 4th grade, and 26000 student ratings were collected in 2018 and 25938 student ratings were collected in 2019.

- Student level information is strictly anonymous. However, what is known is that participating districts in Missouri are diverse, serving both high- and low-income students in urban, suburban, and rural areas. Many serve very low-income, rural White students in the nations' poorest counties. Overall, students in NEE school districts are 80% White, non-Hispanic and 49% are eligible for free or reduced-price meals. The state averages are 73% and 50% respectively, suggesting NEE districts are fairly representative of the state.

Procedures

Online delivery

- End of the school years
- 2017-2018
- 2018-2019

Student ratings

- Unique access code for each teacher
- Anonymous
- Trained adult proctor
- Standard administration scripts

Data screening

- High turnover: at least a month
- Unrealistic time: ± 3 SD from the mean
- Attention check: 3 evenly distributed items

Data retrieval

- End of 2019
- Manually downloaded from the NEE database
- IRB approval
- District consents

Data processing

- Aggregation at teacher level
- Psychometric properties
- Analysis

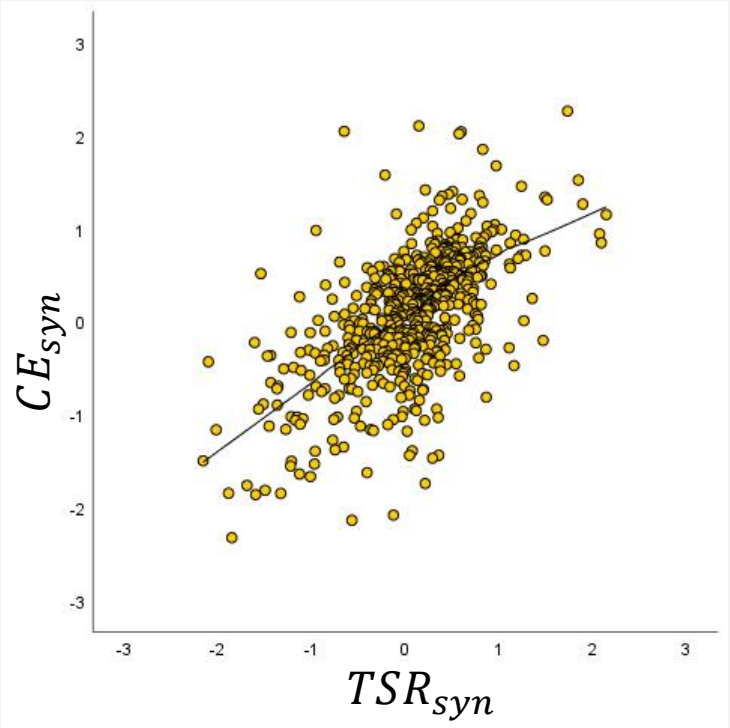
Application Software

- SPSS macros
  - Wiedermann, W., & Li, X. (2018). **Direction dependence analysis: Testing the direction of effects in linear models with implementation in SPSS**. *Behavior Research Methods*, 50 (4), 1581-1601.
- SPSS Custom Dialogue
  - Li, X., & Wiedermann, W. (*In press*). Using SPSS to test the direction of effects in reversible linear models: Introduction into DDA SPSS custom dialogue box with a worked example. In W. Wiedermann, D. Kim, E. Sungur & A. von Eye(Eds.) Direction Dependence in Statistical Models: Methods of Analysis (Book Chapter)
- R scripts/ Python
- Software and manual are downloadable at ddaproject.com

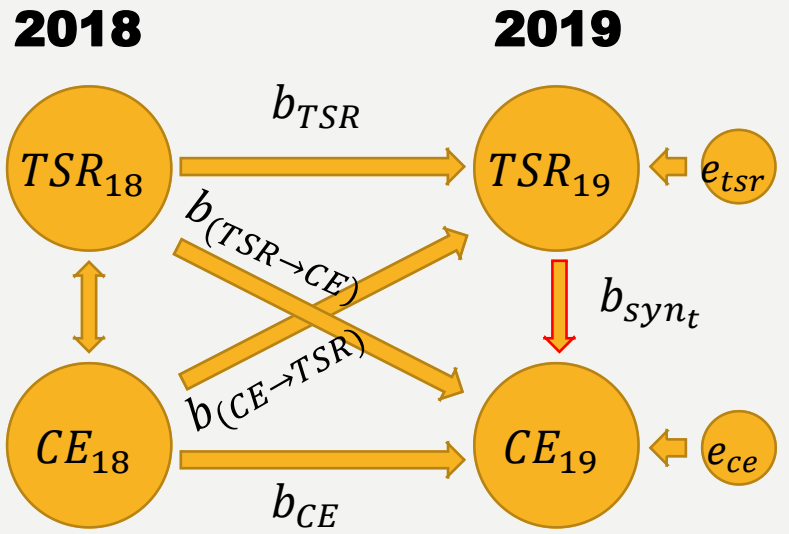
Results: Independence Tests

Competing Models	R^2_{adj}	Breusch–Pagan test		Hilbert-Schmidt Independence Criterion	
		χ^2	p-value	HSIC	p-value
$TSR_{syn} \rightarrow CE_{syn}$	.374	4.114	.128	.058	.106
$CE_{syn} \rightarrow \mathrm{TSR_{syn}}$	.378	12.478	.002	.275	< .000

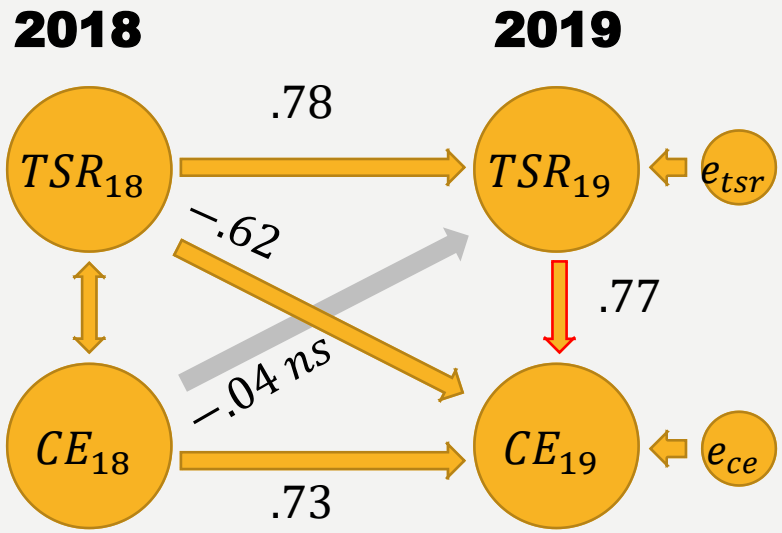
Note:
1. TSR_{syn} and CE_{syn} are the synchronous effect terms controlling both autoregressive and cross-lagged effects, as well as a potential confounder: teachers' years of experience.
2. The p value of HSIC is based on 500 bootstrap samples.



Conclusion:
$TSR_{syn} \rightarrow CE_{syn}$

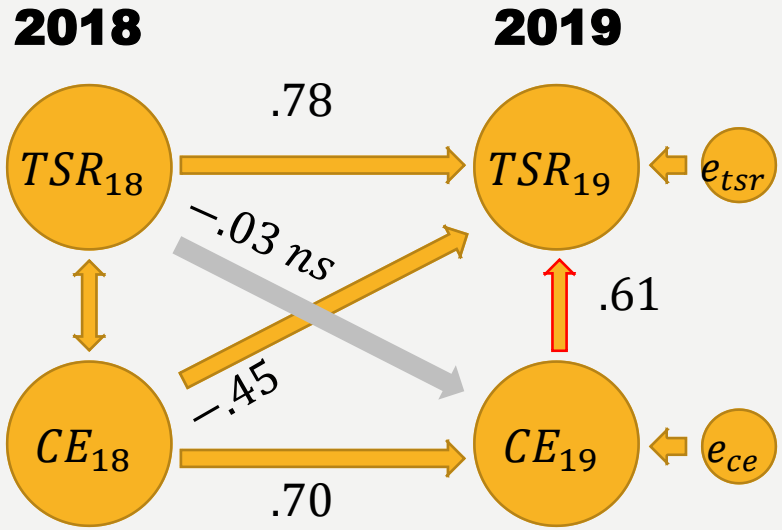


Results: Cross-lagged panel analysis

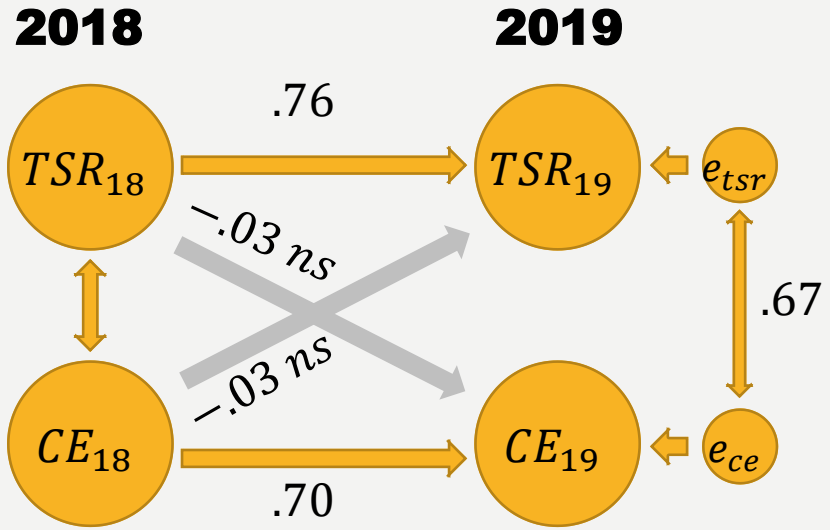


- Synchronous effect
  - Halo effect
  - Necessary platform (Jimenez & Rose, 2010; Klem & Connell, 2004))
  - Teacher motivation (Spilt, Koomen, & Thijs, 2011)
- Cross-lagged effect
  - Engage in different ways
  - Other mechanism

Alternative Erroneous Models



Model DM.



Model ZS.

Fit Statistics / Models	CFI	TLI	RMSEA	SRMR	AIC
Current Model	.974	.965	.073	.051	472.49
Model DM.	.974	.965	.073	.051	472.49
Model ZS.	.974	.965	.073	.051	472.49

Thank You!!

- For more information about predictor-residual independence tests and other direction dependence tests, please go to ddaproject.com

- For more information about Network for Educator Effectiveness, please go to neeadvantage.com