Title: The Effects of Cognitive Strategy Instruction on Math Problem Solving of Seventh-Grade Students of Varying Ability

**Background/Context:** Mathematical problem solving is a complex, recursive cognitive activity involving multiple cognitive processes and two primary phases that assume a working understanding of these processes: problem representation and problem execution (Mayer, 1998; Polya, 1945/1986). Students with learning disabilities (LD) characteristically are poor problem solvers. They generally lack knowledge of problem solving processes, particularly those necessary for representing problems, and, therefore, need to be taught explicitly how to apply them when solving math word problems (Montague & Applegate, 1993). *Solve It!*, the intervention investigated in this school-based efficacy study, helps teachers understand the types of processes and strategies proficient problem solvers use, why many students are ineffective problem solvers, and how they can teach students to be successful problem solvers (Montague, 2003).

*Solve It!* incorporates the cognitive processes essential to problem solving and develops in students the ability to apply these processes when solving math word problems. It places particular emphasis on teaching students the processes and strategies needed to represent mathematical problems. Students learn to paraphrase problems by putting the problem into their own words and visualize problems by developing schematic representations. Problem representation leads to problem execution wherein successful problem solvers monitor themselves as they develop and carry out a logical solution plan. Given that metacognition or self-regulation plays a central role in problem solving, *Solve It!* incorporates self-instruction, self-questioning, and self-monitoring. In sum, *Solve It!* is a comprehensive strategic routine consisting of seven cognitive processes (read, paraphrase, visualize, hypothesize, estimate, compute, and check) and corresponding self-regulation strategies (self-instruction, self-questioning, and self-monitoring) in the form of a SAY, ASK, CHECK procedure. The ultimate goal of instruction is to have students internalize the cognitive processes and self-regulation strategies so that they are used automatically during problem solving. Figure 1 presents the *Solve It!* routine.

An explicit instruction model is the basic procedural approach for teaching students how to use *Solve It!*. Explicit instruction incorporates validated teaching strategies such as cueing, modeling, rehearsal, and feedback. This approach, characterized by structured, organized lessons; appropriate cues and prompts; guided and distributed practice; immediate and corrective feedback on learner performance; positive reinforcement; overlearning; and mastery allows teachers to adapt the teaching routine and tailor instruction to accommodate the strengths and weaknesses of students (Montague, 2003; Montague, Warger, & Morgan, 2000).

**Purpose/Objective/Research Questions/Focus of Study:** The purpose of this intervention study was to improve the mathematical problem solving of middle school students with a particular focus on students with LD by providing general education math teachers with a research-based instructional program that explicitly teaches students how to solve mathematical word problems. The poor mathematics performance of students in our nation’s schools has been demonstrated consistently on state, national, and
international mathematics tests. Mathematical problem solving remains a major concern that poses significant challenges, particularly for students with LD.

In the present study, Solve It!, although originally developed specifically to improve mathematical problem solving for students with LD (Montague, 1992; Montague, Applegate, & Marquard, 1993; Montague & Bos, 1986), was investigated in the context of general education math classes that included low-achieving (LA) and average-achieving (AA) students as well as students with LD. Three research questions guided the present study:

1. What are the effects of the intervention on progress over time of seventh-grade students receiving Solve It! instruction compared with students receiving typical classroom instruction as measured by curriculum-based measures (CBM)?
2. Are there differential effects of the intervention for students of varying ability levels (i.e., students with LD, LA students, AA students)?
3. What are the effects of the intervention on performance on FCAT-style problems over time for students of varying ability levels?

Population/Participants/Subjects: Initially, 20 matched pairs of middle schools were recruited from the Miami-Dade County Public Schools (M-DCPS), the fourth largest school district in the nation. Schools were matched on FCAT performance level and socio-economic status. School performance level was operationalized as the Florida Department of Education’s assigned school grade (A, B, C, D, or F) based on FCAT performance. School level SES was operationalized as the percentage of students who qualified for free or reduced lunch. Attrition during the first two months of the project resulted in 34 participating schools/teachers. All students from the teachers’ inclusion math, intensive math, general math, and pre-algebra class periods had equal opportunity to participate in the study. All students in the intervention schools were provided instruction, but data were collected only from students who returned consent forms (n = 1,079).

Intervention/Program/Practice: Solve It! includes a detailed instructional guide, scripted lessons, class charts, student cue cards, and multiple practice problems. All intervention materials including class sets of FCAT-style practice problems were provided for the school year. The scripted lessons incorporate explicit instructional procedures for helping students acquire, apply, maintain, and generalize problem-solving processes, strategies, and skills (see Montague, 2003). Following a three-day professional development workshop for the intervention teachers, the intervention began in October and consisted of three days of intensive instruction followed by weekly problem-solving practice sessions using 196 problems drawn from the district curriculum and FCAT practice manuals. The teachers were given print copies of the problems as well as an electronic file of problems. Scripts were provided for the initial three lessons and the practice sessions. The first author modeled a practice session for the intervention teachers at least once during a practice session. Students in the comparison group received only typical classroom instruction or “business as usual,” which followed the district mandated “pacing guide” that linked the textbook to the Florida State Sunshine Standards. To ensure comparable practice time between the intervention and comparison groups, comparison group teachers were asked to focus on word problem solving for at least one class period per week using FCAT problems from the text and FCAT practice tests.
Students in both groups were allowed to use calculators during practice and testing sessions.

Seven CBM of math problem solving consisting of 30 unique test items selected from the *Solve It!* manual (Montague, 2003) were calibrated using Item Response Theory methods (i.e., the Rasch model; Osterlind, 2000) to achieve equivalent difficulty level. The internal consistency of the measures ranged from .67 to .80. Each of the seven measures consisted of 10 one-, two-, and three-step textbook-type math problems. The problems did not require specific or unique mathematical knowledge or concepts; rather, they required students to perform the four basic operations using whole numbers or decimals. These curriculum-based measures were administered to each intervention teacher’s participating math classes seven times, specifically, prior to the intervention (baseline) and then approximately monthly for the remainder of the school year (progress monitoring). The measures were administered four times to the comparison group classes, specifically, prior to the intervention (baseline) and then at the third, fifth, and seventh administrations. Comparison group teachers were observed during fall and spring using narrative reporting. Beginning with the third administration of the CBM, the measure was scored, raw score data were entered into the database, and graphs for individual consented students in the intervention group were generated. These graphs provided feedback to teachers and students regarding student progress over time.

Level of treatment fidelity and inter-rater agreement were averaged across the observations separately for the initial three lessons and the practice sessions. Percentages were calculated by dividing number of agreements by agreements plus disagreements multiplied by 100. For the observations of the initial three lessons, fidelity of treatment averaged 97% (range 90% - 100%) and inter-rater agreement averaged 99%. For the weekly practice sessions, fidelity of treatment averaged 93% (range 77% - 100%) and inter-rater agreement averaged 99%.

**Research Design/Analyses:** The data were consistent with a 3-level model where repeated measures (level-1) were nested within students (level-2), and students were nested within schools (level-3). The first analysis was an unconditional MLM that partitioned problem-solving ability variation into the three levels. This analysis indicated that individual differences between students (i.e., level-2) accounted for nearly 30% of the variability, and mean differences between schools (i.e., level-3) accounted for 20% of the problem solving variation. The remaining 50% of the variability reflected within-person score differences. Having established the presence of variation at all three levels of the data hierarchy, we added the level-1 temporal predictor that quantified the timing of the repeated measures variables. This variable was expressed as the number of months since the first day of October. Centering assessment time in this manner expressed the intercept of the MLM as the baseline problem-solving score. Next, we performed a series of analyses to determine whether the growth rates varied across individuals at level-2 and across schools at level-3. After allowing the average growth rates to vary across schools, the between-person variance in growth rates was non-significant. Consequently, we removed this random effect and proceeded with a model that allowed for baseline score differences among individuals within a school, baseline mean differences between schools, and variation in the average monthly growth rates between schools.

**Findings/Results:** Results indicated that the intervention and comparison schools started at a similar level but the intervention group increased at a rate that greatly
exceeded that of the comparison group. Expressed relative to the average within-school standard deviation, this mean difference corresponded to a large standardized effect size (nearly one SD difference; Cohen, 1988. The final set of analyses explored whether the effect of the intervention differed across (i.e., was moderated by) ability groups (students with LD, LA students, AA students). The students with LD had problem-solving scores that were lower than those of the AA students by approximately five points. The LA students had scores that were approximately one point lower than the AA students, on average. From a practical perspective, the model comparisons indicated that, although the students with LD had lower problem-solving scores, the intervention had a uniform impact across ability levels.

**Conclusions:** Progress monitoring as determined by performance on the CBM over a school year showed that seventh-grade students who received the intervention made significantly greater growth in mathematical problem solving than the comparison group students who received typical classroom instruction. Thus, the findings were positive and support the efficacy of Solve It! as an intervention to improve math problem solving for middle school students. Specifically, the results indicated that students who received Solve It! instruction across ability groups, including students with LD, showed greater growth in math problem solving over the school year than students in the comparison group. That is, the intervention group performed statistically significantly better than the comparison group on the CBM across time, and the difference was also practically significant with a difference between groups of nearly one standard deviation, a large effect size. Further, the intervention appeared to have the same impact for students across ability groups as they improved at the same rate over time, although, as one would expect, AA students performed better initially and continued that advantage. In contrast, students with LD consistently scored lower than both LA and AA students. However, a most encouraging finding was that students with LD outperformed all ability groups in the comparison group, even the AA students, by the end of the school year. All ability groups improved at the same rate. That is, they started at approximately the same level as their comparison group peers and improved significantly over time. In contrast, the performance of their comparison group peers on the CBM remained virtually unchanged across the school year. Improving students’ math problem solving should enable them to perform better overall in mathematics, which should have a positive impact on their grades, success in school, graduation rate, and, ultimately, on post-secondary outcomes.
Figure 1.

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**Solve It! - Math Problem Solving Processes and Strategies**

**READ** (for understanding)
Say: Read the problem. If I don’t understand, read it again.
Ask: Have I read and understood the problem?
Check: For understanding as I solve the problem.

**PARAPHRASE** (your own words)
Say: Underline the important information. Put the problem in my own words.
Ask: Have I underlined the important information? What is the question?
Check: That the information goes with the question.

**VISUALIZE** (a picture or a diagram)
Say: Make a drawing or a diagram. Show the relationships among the problem parts.
Ask: Does the picture fit the problem? Did I show the relationships?
Check: The picture against the problem information.

**HYPOTHESIZE** (a plan to solve the problem)
Say: Decide how many steps and operations are needed. Write the operation symbols (+, −, x, and /).
Ask: If I …, what will I get? If I …, then what do I need to do next? How many steps are needed?
Check: That the plan makes sense.

**ESTIMATE** (predict the answer)
Say: Round the numbers, do the problem in my head, and write the estimate.
Ask: Did I round up and down? Did I write the estimate?
Check: That I used the important information.

**COMPUTE** (do the arithmetic)
Say: Do the operations in the right order.
Ask: How does my answer compare with my estimate? Does my answer make sense? Are the decimals or money signs in the right places?
Check: That all the operations were done in the right order.

**CHECK** (make sure everything is right)
Say: Check the plan to make sure it is right. Check the computation.
Ask: Have I checked every step? Have I checked the computation? Is my answer right?
Check: That everything is right. If not, go back. Ask for help if I need it.

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References