Modeling Mediation: Causes, Markers, and Mechanisms

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Sea Change in Ed Research!

- Role of Experiments
  - Cluster randomized trials
  - Smart designs, Well powered!
- Sound Quasi-Experiments
  - Regression Discontinuity
  - Propensity score adjustment
  - Instrumental variables

What Experiments Can Tell Us

- Clear answer to sharp causal questions:
  - Average effect of assignment to treatments
  - If high compliance, average effect of an intervention
  - For whom does the intervention work?
  - In what settings does it work?

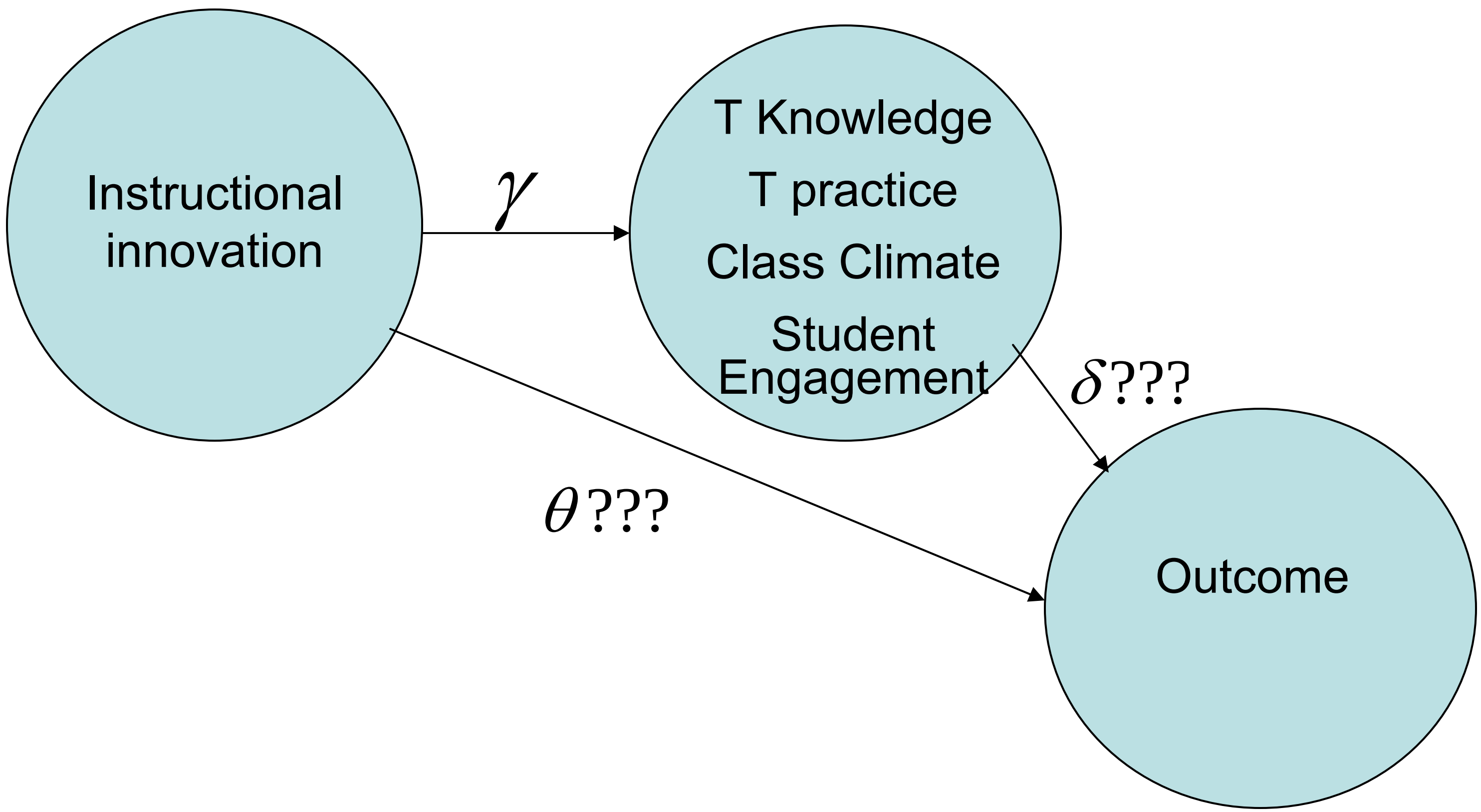
What about “How?” and “Why?”

- Example: IES study of pre-school curricula
  - Tells us which, on average, work better
  - But why
    - More time of academic instruction?
    - Better content?
    - Increase in teacher skill?
    - Compensate for lack of teacher skill?
    - Heighten child engagement?

Questions of Mediation

- In lab experiments, conduct a subsequent "factorial" randomized study
  - E.g., curricula by instructional time
- Very costly in large-scale field studies

Major Progress in Measurement



Some Major Progress

- We can ask
  - Was the treatment implemented
  - Did the treatment assignment affect practice?
  - Did the treatment increase student engagement

- These help interpret average causal effects

More challenging

- Did the treatment affect *Y* by
  - Increasing teacher knowledge?
  - Increasing teacher skill?
  - Improving class climate?
  - Increasing student engagement?
- These are questions of mediation

Models for Mediation

- Conventional methods under scrutiny
- Alternatives
  - Natural direct and indirect effects
  - Controlled direct effects
  - Direct Effects within Principal Strata
  - Instrumental variables for indirect effects
- Each requires assumptions
- Methods of Estimation not widely accessible

Punch Line

- Each approach answers a different question
- So let’s think hard about our questions

Organization of Talk

Consider questions about

- Demographic markers that predict access to treatments that predict outcomes
- Treatments that predict access to future treatments that predict outcomes
- Treatments that affect surrogate markers that predict outcomes

In light of these questions consider

- Oaxaca Decomposition
- Natural Direct Effects
- Controlled Direct Effects
- Principal Stratification
- Instrumental Variables

Reflect on the larger project: finding mechanisms

“All-purpose” mediation model

(Blau and Duncan, 1965; Baron and Kenny and Baron 1986)

Predictor	*Mediator*	*Outcome*
Demographic marker (parent SES)	Treatment (years of ed.)	Outcome (earnings)
Treatment (new curriculum v old)	Mediating treatment (quality of instruction)	Outcome (learning)
Treatment (pre-school program)	Surrogate marker (early vocab)	Outcome (Reading comp)

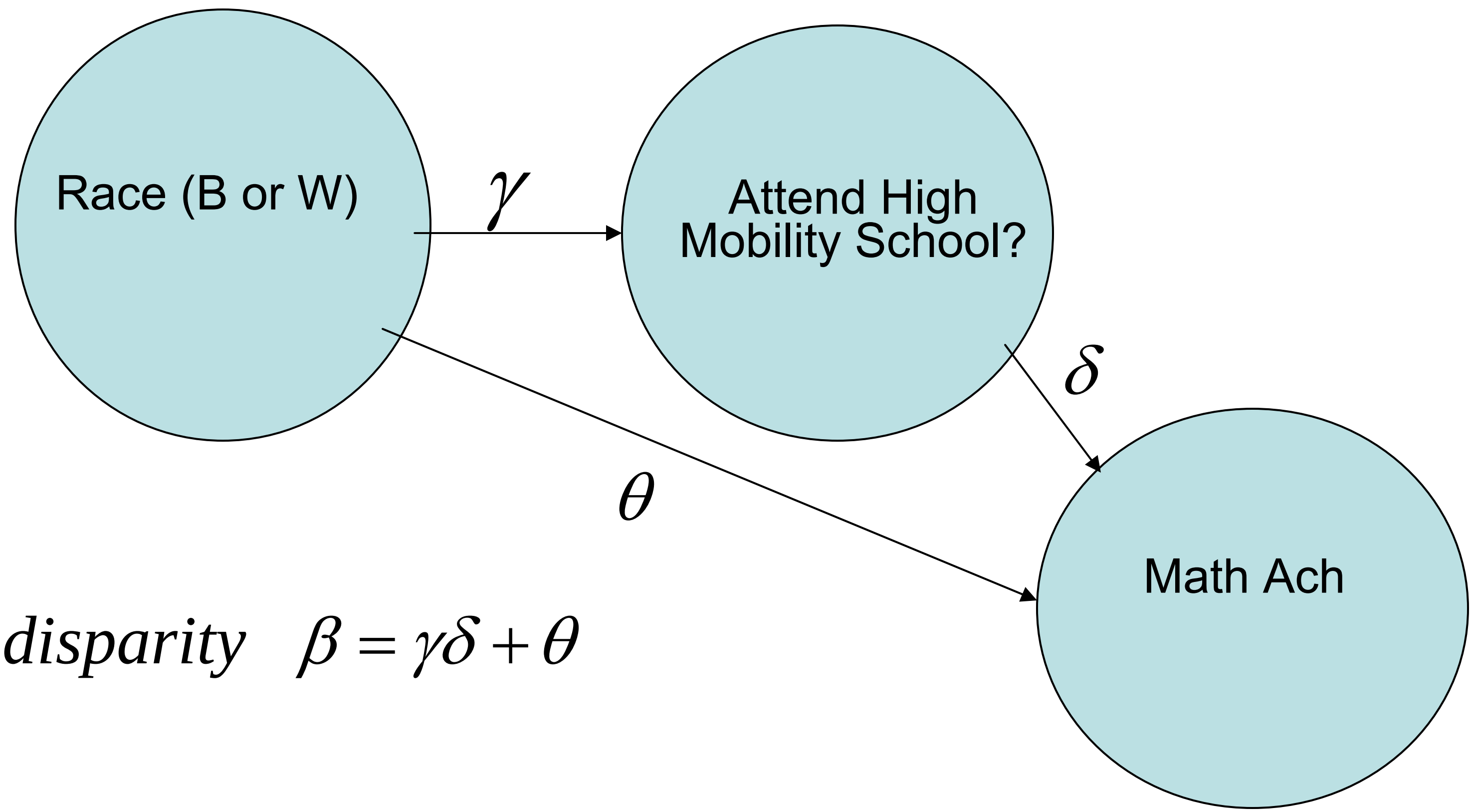
Start with Case 1

Demographic Marker-->Treat-->Y

Example:

Does school mobility explain B-W test score gap? (Raudenbush, Jean, Art, 2011)

Conventional Model



$$Racial\ disparity \quad \beta = \gamma\delta + \theta$$

But we found differential effects!

$$\gamma = (\phi_b - \phi_w) = \text{differential exposure}$$

$$\delta_b - \delta_w = \text{differential vulnerability}$$

$$\textit{Oaxaca decomposition}:$$

$$\begin{aligned}\beta = (\phi_b - \phi_w)\delta_b \qquad & \left(\textit{indirect path}\right) \\ + \theta_{\text{low}} + \phi_w(\delta_b - \delta_w) \quad & \left(\textit{direct path}\right)\end{aligned}$$

Decomposition is not unique

$$\beta = (\phi_b - \phi_w)\delta_b \quad \left(indirect\ \ path\right)$$
$$+\theta_{\mathrm{low}} + \phi_w(\delta_b - \delta_w) \quad \left(direct\ \ path\right)$$

$$= (\phi_b - \phi_w)\delta_w \quad \left(indirect\ \ path\right)$$
$$+\theta_{\mathrm{low}} + \phi_b(\delta_b - \delta_w) \quad \left(direct\ \ path\right)$$

Alternative expression

$$
\begin{aligned}
\beta = {} & (\phi_b - \phi_w)\delta_b \quad \left(indirect \;\; path\right) \\
& + \phi_w \theta_{\mathrm{high}} + (1 - \phi_w)\theta_{\mathrm{low}} \quad \left(direct \;\; path\right)
\end{aligned}
$$

Case 1 is easy!

- We have to estimate only 1 causal effect for Blacks and 1 for Whites
- Assume ignorable assignment to treatment within race
- Eg: How much does Class size reduction reduce the B-W gap (Krueger and Whitmore, 19xx)

Case 2

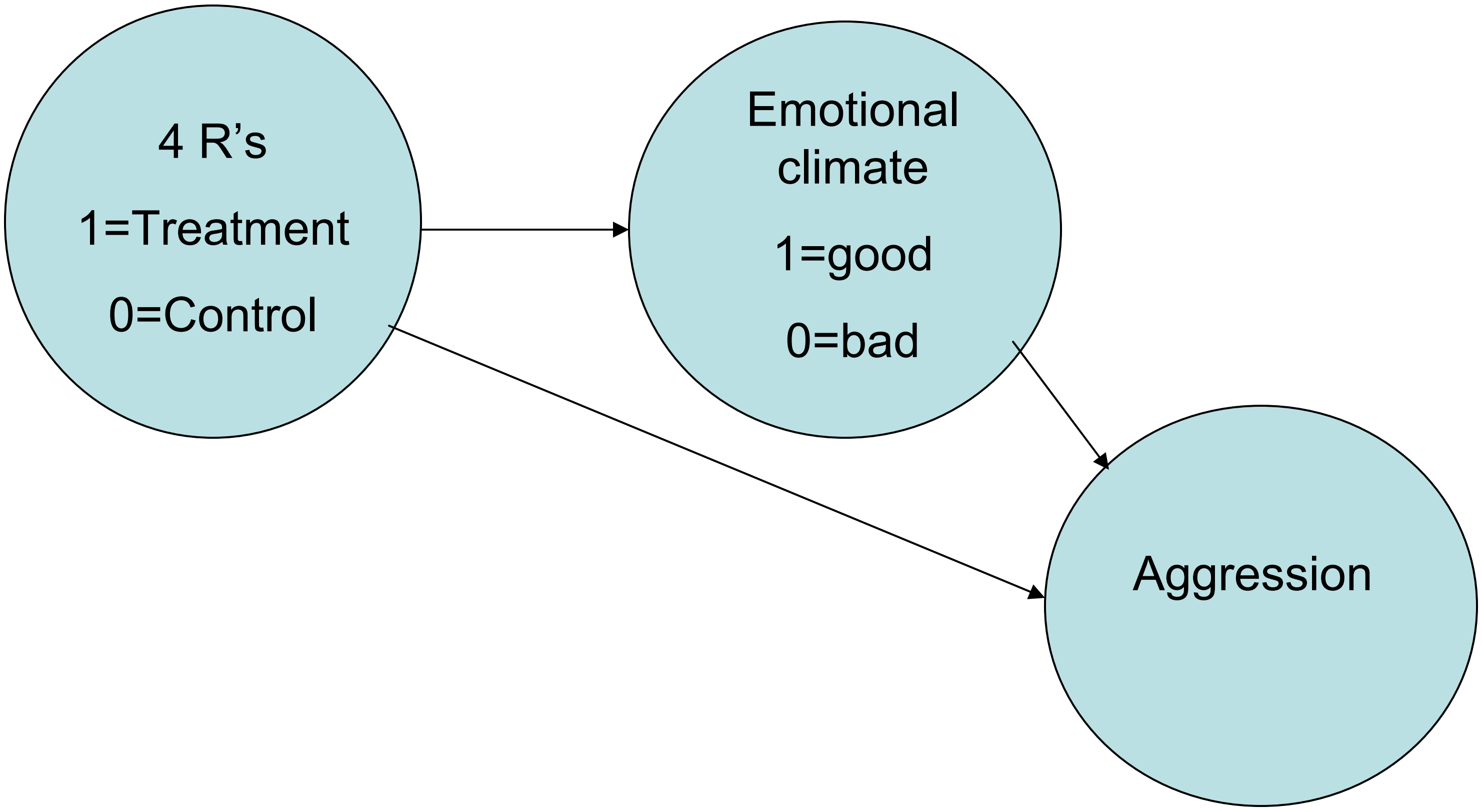
Treatment → Mediating Treatment → Y

Example:

Does a new curriculum reduce aggression by improving the emotional climate of the classroom?

(VanderWeele, Hong, Jones, and Brown, 2011)

Assignment to 4Rs Induces Better Climate

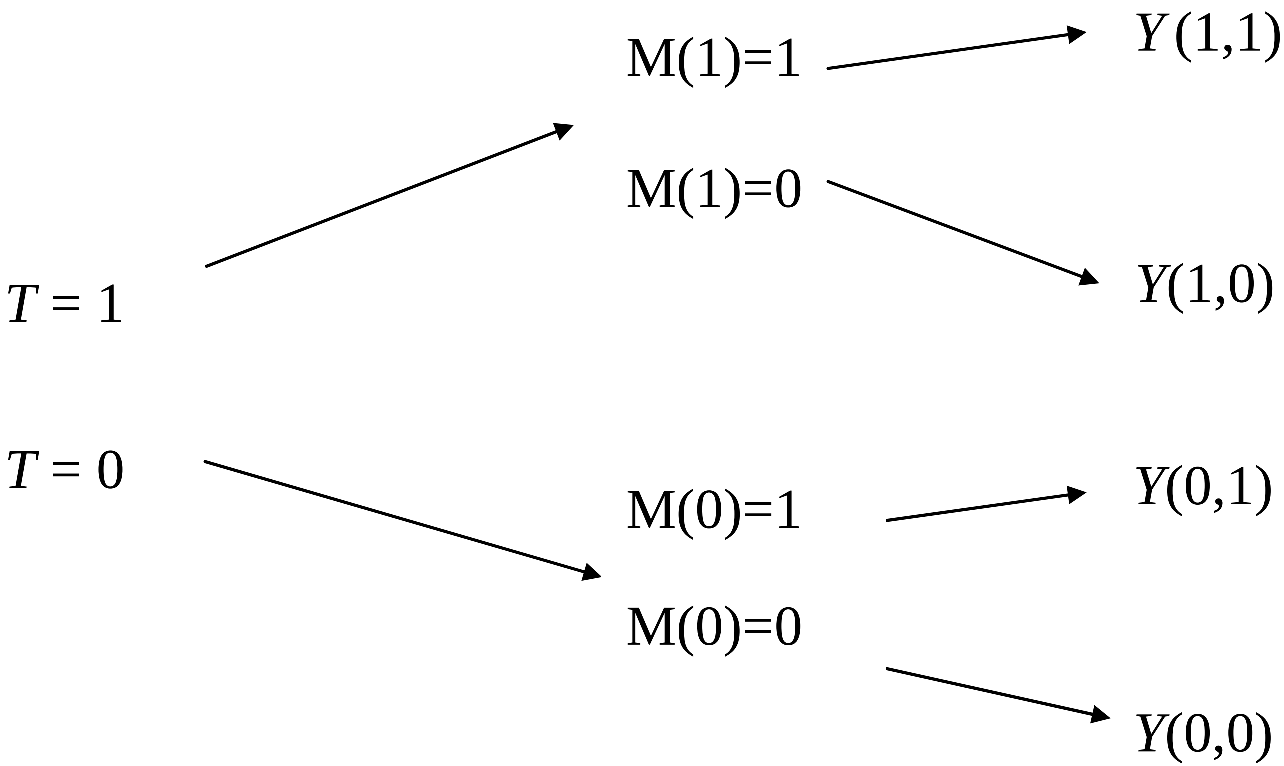


Must estimate causal effects of a sequence of two treatments!

The first (randomized) treatment increases the probability of exposure to the second (non-randomized) treatment.

Assumptions are subtle and strong!

Person-specific Potential Outcomes



Person-specific Causal Effects

- Effect of *T* on *M*

$$\Gamma = M(1) - M(0)$$

- Effect of *M* on *T*

$$\Delta_1 = Y(1,1) - Y(1,0) \quad \text{effect if } T = 1$$

$$\Delta_0 = Y(0,1) - Y(0,0) \quad \text{effect if } T = 0$$

- *Direct Effect of T on Y*

$$\Theta_1 = Y(1,1) - Y(0,1) \quad \text{effect if } M = 1$$

$$\Theta_0 = Y(1,0) - Y(0,0) \quad \text{effect if } M = 0$$

Natural Direct and Indirect Effects

- Total Effect: person-specific "Oaxaca Decomposition!"

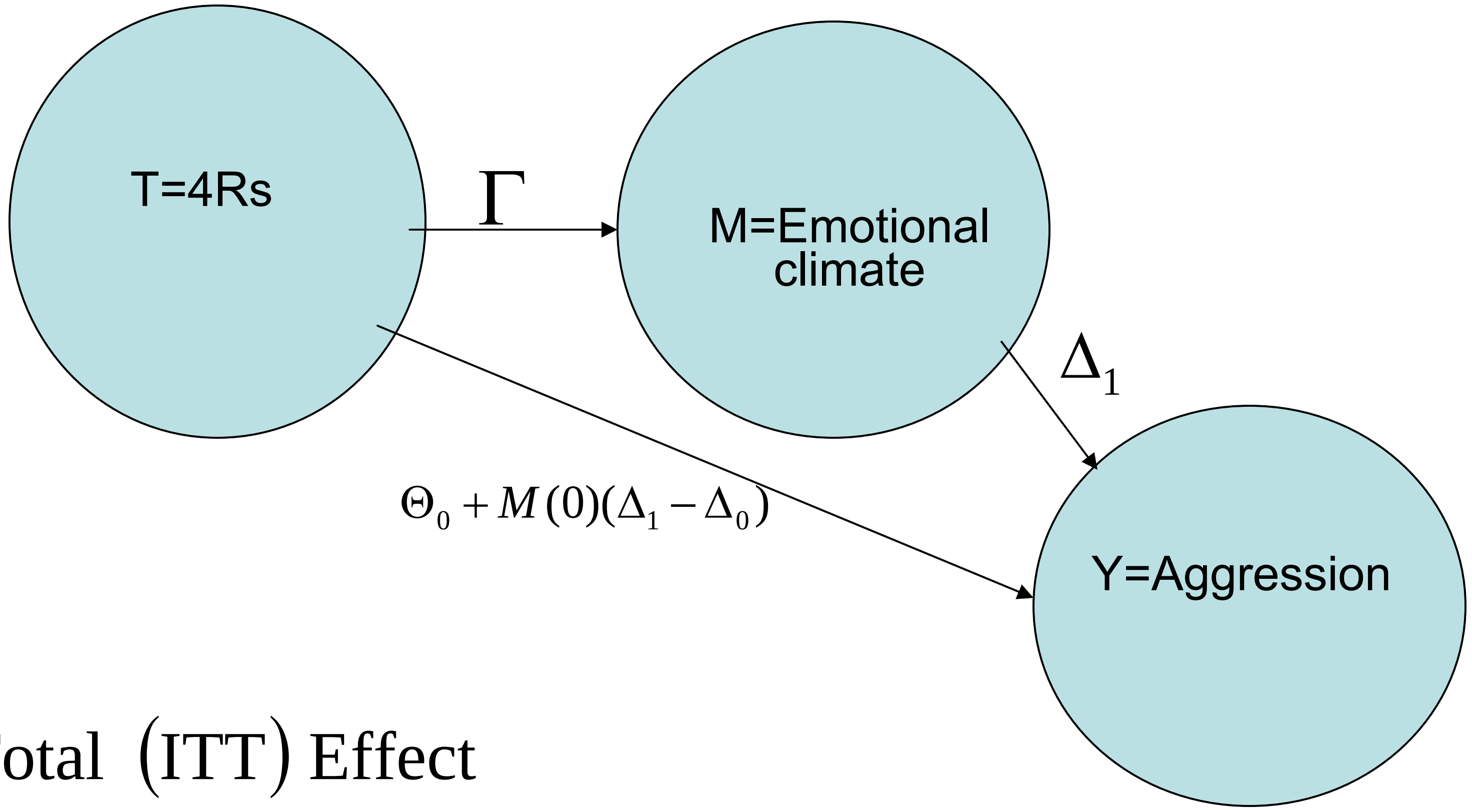
$$B = Y(1, M(1)) - Y(0, M(0))$$

$$\begin{aligned} &= Y(1, M(1)) - Y(1, M(0)) \quad \text{natural indirect effect} \\ &\quad + Y(1, M(0)) - Y(0, M(0)) \quad \text{natural direct effect} \end{aligned}$$

$$= \Gamma\Delta_1 \quad \textit{natural indirect effect}$$

$$+ \Theta_0 + M(0)(\Delta_1 - \Delta_0) \quad \textit{natural direct effect}$$

Person-specific Causal Model



$$\text{Total } (\text{ITT}) \text{ Effect}$$

$$B = \Gamma\Delta_1 + \Theta_0 + M(0)(\Delta_1 - \Delta_0)$$

But what is the *average* causal effect?

$$E(B) = \beta = E[\Gamma\Delta_1 + \Theta_0 + M(0)(\Delta_1 - \Delta_0)]$$

$$= \gamma\delta_1 + Cov(\Gamma, \Delta_1)$$
$$+ \Pr(M(0) = 1)(\delta_1 - \delta_0) + Cov(M(0), \Delta_1 - \Delta_0)$$

$$= \gamma\delta_1 + \theta_0 + \Pr(M(0) = 1)(\delta_1 - \delta_0)$$

(if we assume away covariances)

$$= \gamma\delta + \theta$$

(if we assume away differential effect of mediator)

Summary on Natural Direct and Indirect Effects

- Assume ignorable assignment to T
- Assume ignorable assignment to M within levels of T
- Assume away troublesome covariances
- Estimate via a regression method (Peterson et al.) or a non-parametric weighting method (Hong)
- Assume in addition no differential effects and linearity and use conventional methods!

Controlled Direct Effects

- Estimate

$$\theta_0 = E[Y(1,0) - Y(0,0)]$$
$$\theta_1 = E[Y(1,1) - Y(0,1)]$$

- Still need ignorable assignment to T
- Ignorable assignment to M given T
- Eliminates Troublesome covariances!
- But no estimation of indirect effect

Instrumental Variables

- Recall $B = \Gamma\Delta_1 \quad natural\ \ indirect\ \ effect$
  $+\Theta_0 + M(0)(\Delta_1 - \Delta_0) \quad natural\ \ direct\ \ effect$

- If we assume direct effect=0, we have

$$B = \Gamma\Delta \qquad \left(\Delta = Y(M(1) - Y(M(0))\right)$$

- Population-average effect

$$E(B) = \beta = E(\Gamma\Delta) = \gamma\delta + Cov(\Gamma, \Delta)$$

- Assume away covariance $\beta = E(\Gamma\Delta_1) = \gamma\delta_1$

- Assume $\gamma \neq 0$, we have $\beta / \gamma = \delta_1$

Summary on Instrumental Variables

- Assume ignorable assignment to *T*
- Assume no direct effect ("exclusion restriction")
- Assume away troublesome covariance
- Assume T affects M $(\gamma \neq 0)$

Case 3

Treatment → Surrogate marker →Y

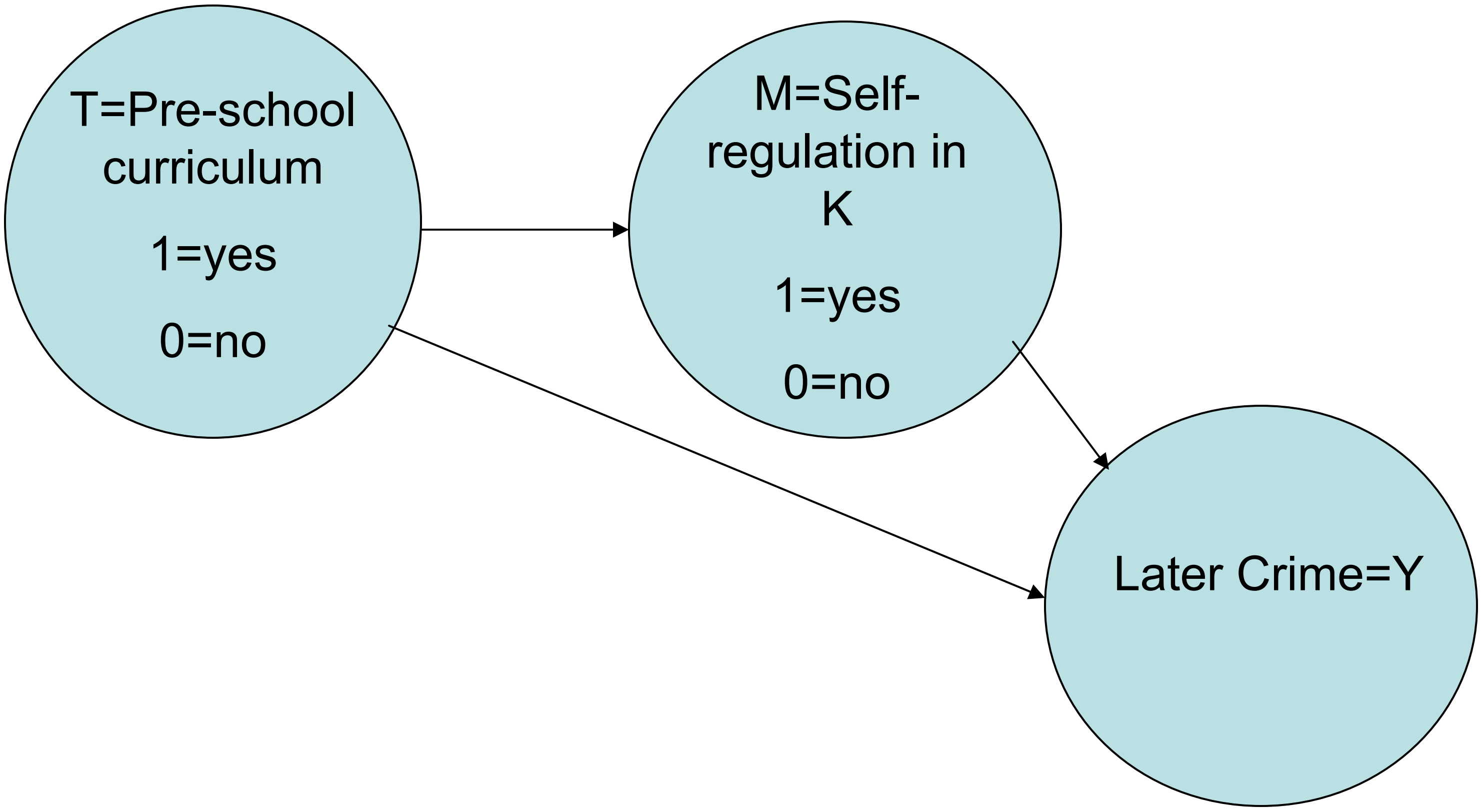
Example:

Does a new pre-school curriculum increase self regulation in K and hence reduce adolescent crime?

Mediator “stands in” for outcome=crime

Mediator is not a treatment

Case 3


T=Pre-school curriculum
1=yes
0=no
M=Self-regulation in K
1=yes
0=no
Later Crime=Y

Principal stratification

	M(1)	M(0)	Total Effect	Direct Effect
Compliers	1	0	Y(1,1)-Y(0,0)	----
Always-takers	1	1	Y(1,1)-Y(0,1)	θ_{AT}
Never-takers	0	0	Y(1,0)-Y(0,0)	θ_{NT}
Defiers	0	1	Y(1,0)-Y(0,1)	----

Summary on Principal stratification

- How a child will respond to intervention is a pre-treatment characteristic of that child
- Causal effects defined only within strata
- Direct effects defined for strata of children whose M is not changed by T.
- Strata are not observed, must be modeled
- No sharp point estimate of causal effects
- Seems most plausible when M is a surrogate marker rather than a second treatment

Summary

Case 1 (Demographic marker→Treat→Y)
- Use Oaxaca decomposition

Case 2 (Treat1→Treat 2→Y)
- Natural direct and indirect effects
- Controlled direct effects
- Instrumental variables

Case 3 (Treat→S. Marker→Y)
- Principal stratification

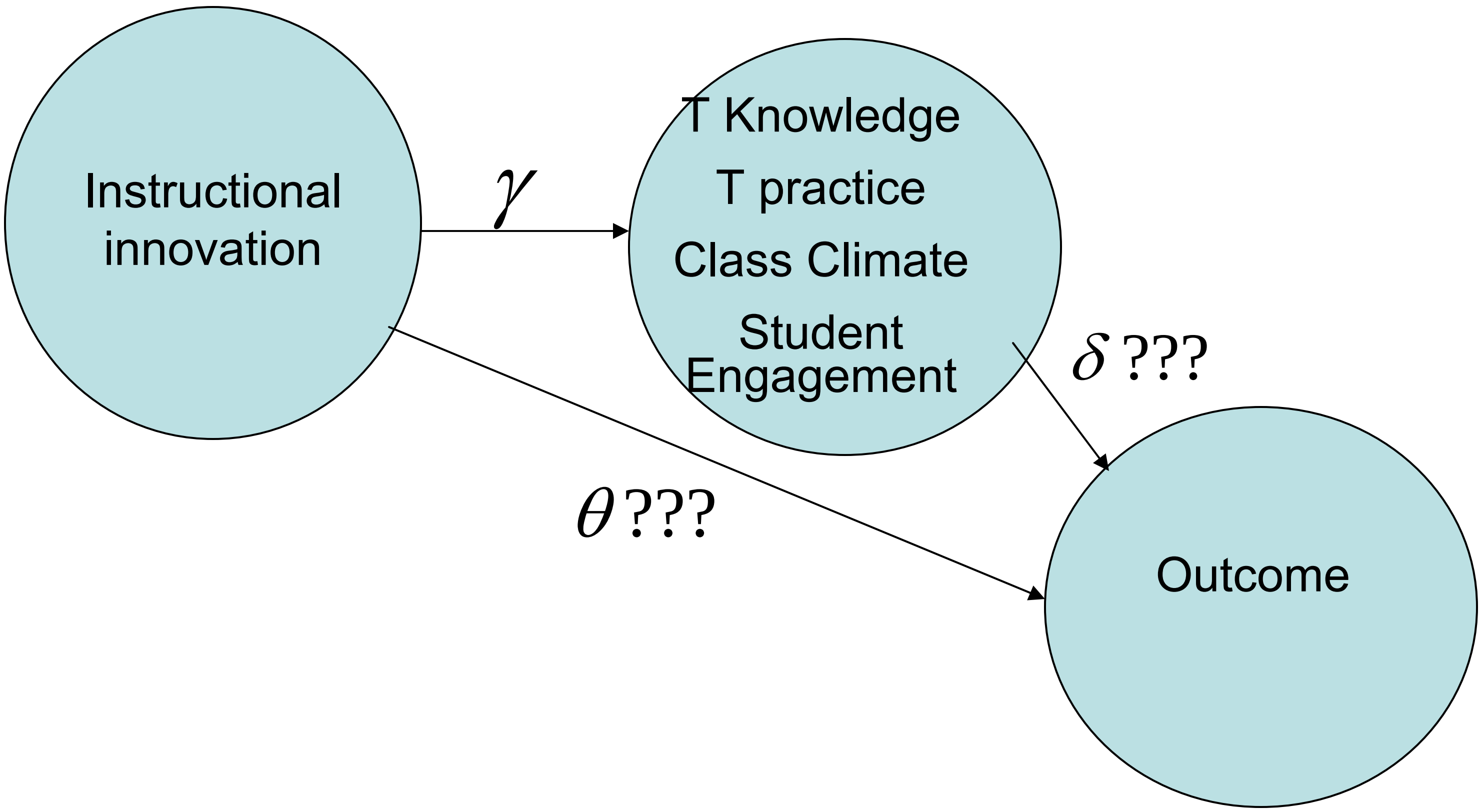
Conclusion

- Each method makes pretty strong assumptions
- The conventional "SEM" makes very strong assumptions
- We should pursue:
  - The method that best suits the question
  - That makes weakest assumptions
- We should state and evaluate the assumptions

However,…

- The project of discovering mechanisms and explaining effects requires a broad, multidisciplinary research agenda

Study “one arrow at a time”



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