Title: Modeling Intervention Effects on Social Networks in Education Research
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Background/Context

In many education interventions, the social network within each school serves as an excellent intermediate outcome variable. Many whole school initiatives encourage some type of social structural change, whether it is an increase in collaboration, distribution of leadership or a push toward small learning communities, all of which can be detected through changes in a social network.

A social network is defined to be a collection of individuals or actors and the ties among them (Kolaczyk, 2009). These ties represent any number of relationships; teacher collaboration, advice-seeking, friendship, trust, etc. The value of these ties can have both direction and magnitude. Teacher $i$ seeking advice from $j$ is a different type of relationship than teacher $j$ seeking advice from teacher $i$; moreover, an advice-seeking tie may be qualitative, seeks advice or does not seek advice, or could be quantitative in nature, the value or amount of the advice received.

There is limited research studying social networks in education effectiveness research but existing work does suggest that teacher network structure can both influence an intervention as well as be affected by an intervention. Frank et al. (2004) and Penuel et al. (2006) found teachers who were well-connected to resources helped influence reform initiatives in a positive way. By the same token, Coburn and Russell (2008) claim that policy decisions influenced the structure of the teacher networks, and Weinbaum et al. (2008) posit that teacher networks in schools involved in reform initiatives had more connected structure than those networks not involved in such initiatives. Finally, Penuel et al. (2010) caution that school-wide initiatives can only affect so much change if the teacher network structure remains unchanged.

Purpose

The purpose of this paper is to introduce novel statistical methodology for modeling causal intervention effects on social networks. No such methodology currently exists despite the apparent utility of using social networks as intermediate outcomes in Education research.

While social network data is used occasionally in education research, the methods are usually descriptive. For example, Coburn and Russell (2008) use the total number of ties to compare networks, and both Frank et al. (2004) and Penuel et al. (2006) use the number of ties specific to each teacher. While these statistics can be informative, summary statistics fail to capture the full structure of the network. Furthermore, using network statistics for comparison across networks poses a multiple comparison issue as there is a number of common summary statistics.

A statistical social network model however accounts for the entire network structure, not just isolated summary statistics. Despite the availability of a variety of social network models and software to fit them, these models have rarely been used in education research. Indeed a review of the literature yielded only two instances and both are observational studies. Penuel et al. (2010) and Weinbaum et al. (2008) fit $p^2$ models (Lazega and van Duijn, 1997), a type of exponential random graph model, to study communication among teachers regarding a reform initiative.
There is good reason that social network modeling is not more prevalent in education research; existing models are inadequate for the types of problems studied in education research. Educational interventions often involve more than one school or network and current models address only a single network. Methods to model a collection of school-based social networks, for example, simply have not been studied by either statisticians or methodologists. Moreover, social network models treating the network as an experimental outcome did not exist up to now.

Significance / Novelty of study

We introduce statistical models that can determine causal effects on social networks. The methodology is completely novel despite the apparent utility for social networks as outcomes. The model we will discuss, the Causal Latent Space Model (CLSM), can also model any sample of independent or partially exchangeable multiple networks; thus our methods are applicable to observational studies as well. The CLSM is one of a larger framework of models currently under development by the authors.

The Causal Latent Space Model for Social Networks

A social network is defined to be a collection of actors or individuals and the ties among them. Let $Y_{ij}$ be the value of a tie from individual $i$ to $j$ and $Y$ the collection of all ties. We include directionality from $i$ to $j$ since not all relational data is reciprocal. $Y_{ij}$ could be binary, ordinal or quantitative. For simplicity in this abstract, let us assume that $Y_{ij}$ is binary.

The latent space model for a single network (Hoff et al., 2002) can be written as

$$P(Y|Z, X, \beta) = \prod_{i \neq j} P(Y_{ij}|Z_i, Z_j, X_{ij}, \beta),$$

where $Z = (Z_1, Z_2, ..., Z_n)$ represents the latent positions $Z_i$ of each actor $i$ and $X$ is a matrix of covariates. Thus, the value of a tie between any two actors is independent of every other tie conditional on the latent social space positions of each actor in the network. When $Y_{ij}$ is binary, we write $P(Y_{ij}|Z_i, Z_j, X_{ij}, \beta)$ as a logit,

$$\text{logit}(P(Y_{ij} = 1)) = \beta_0 + \beta_1^t X_{ij} - |Z_i - Z_j|.$$  

To model $K$ exchangeable networks of size $n_k$, we introduce the Multiple Network Latent Space Model. We assume that each network is independent of every network; ties across networks are mutually independent and ties within one network are only independent conditional on the latent social space positions of actors within that network:

$$P(Y_k|Z_k, X_k, \beta_k) = \prod_{i \neq j} P(Y_{ijk}|Z_{ik}, Z_{jk}, X_{ijk}, \beta_k).$$

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As before, we model binary ties in the Multiple Network Latent Space Model using a logit model

\[
\logit(P(Y_{ijk} = 1)) = \beta_0 + \beta_1 X_{ijk} - |Z_{ik} - Z_{jk}|. 
\]

Furthermore, we can consider network specific parameters as fixed or random. For example, we might let \( \beta_k = \beta \) for \( k = 1, \ldots, K \) or we could instead assume a hierarchical Bayes structure, \( \beta_k \sim MVN(\mu_{\beta}, \Sigma_{\beta}) \) for \( k = 1, \ldots, K \). We may impose a similar hierarchical structure with our latent positions. \( Z_{ik} \sim MVN(\mu_{Z_k}, \Sigma_{Z_k}) \) for \( i = 1, \ldots, n_k \) with \( \mu_{Z_k} \sim N(\mu_0, \Sigma_{\mu}) \) and \( \Sigma_{Z_k} \sim Inv - \chi^2(a_0, b_0) \). This allows both independent as well as partially exchangeable structures across networks.

We then extend the Multiple Network Latent Space Model to include intervention effects.

\[
\logit(P(Y_{ijk} = 1)) = \beta_0 + \beta_1 X_{ijk} - |Z_{ik} - Z_{jk}| + \alpha T_k
\]

where \( T_k \) is a network-specific treatment indicator with treatment effect \( \alpha \).

For a single network, there are two common methods for parameter estimation, maximum likelihood and Markov Chain Monte Carlo (MCMC; (Gelman et al., 2007)), and both methods are available in the \texttt{latentnet} package (Krivitsky and Handcock, 2008) in R (R Development Core Team, 2011).

Currently, the Causal Latent Space Model is estimated using MCMC; in particular, we use a Gibbs Sampler with Metropolis updates for each parameter.

**Applicability of Method**

While the CLSM would be best applied to intervention data on social networks, lack of methodology has prohibited such studies. We recognize the importance of assessing our methodology of real data. Thus for the conference paper, we will include an application of our Multiple Network Latent Space Model to observational advice-seeking network data from fifteen schools in our full paper.

For now we demonstrate our model on simulated binary network data. We simulate ties among 10 teachers in 20 schools of which 10 are treatment schools. We assume directed ties, a tie from \( i \) to \( j \) is not necessarily the same as the tie from \( j \) to \( i \). We assume that ties across networks are mutually independent, and ties within each network are mutually independent conditional the latent social space positions of the teachers in that network.

Furthermore, we include a fixed intercept and random slope as parameters in our simulation models. For simplicity, we use the same slope parameter for each school. We chose these particular values of \( \beta_0 \) and \( \beta_1 \) to produce graphs that were neither empty (no ties) or complete (ties between every teacher). We randomly assigned each teacher in each school one of four grades, and \( X_{ij} \) is the tie specific indicator for teaching the same grade. \( T \) is an indicator for treatment and \( \alpha \) is our intervention effect. Finally, we assume a 2-dimensional latent space.

Thus, the probability of a tie from teacher \( i \) to teacher \( j \) in school \( k \) is given by
\[ \eta_{k(ij)} = 2 + 4X_{k(ij)} + |Z_{k(i)} - Z_{k(j)}| + \alpha T_k. \]

Finally we simulate four sets of observed tie data for \( \alpha = 3, \alpha = 2, \alpha = 1, \) and \( \alpha = 0.5 \) as follows:

\[ Y_{ijk} \sim \text{Bernoulli} \left( \frac{\exp(\eta_{k(ij)})}{1 + \exp(\eta_{k(ij)})} \right). \]

To fit each of our four models, we run an MCMC algorithm for 10,000 iterations with a burn-in of 3,000 iterations. We then thin our chain by selecting every 25th sample. Our posterior density plots for treatment effect from each model are given in Figure 1. The true value for the data generating treatment effect is shown as a vertical line on each density plot. Posterior modes and 95% credible intervals for treatment effect from each model are given in Table 1. For each value of \( \alpha \), our model’s posterior mode is within the corresponding 95% credible interval as our estimates for \( \alpha \) are just slightly lower than the true value of \( \alpha \).

Our model is able to detect significant treatment effects from simulated data when the true \( \alpha \) is 3, 2, and 1, despite a small sample of networks (\( n=20 \)) and a small number of teachers within each school (\( t=10 \)). As our 95% credible interval does not exclude 0 when data is generated from \( \alpha = 0.5 \), we do not recover significant treatment effects when \( \alpha = 0.5 \). This is not particularly surprising as a 0.5 increase in log odds translates to a minimal increase in tie probability for most probability values.

Conclusions

We recommend the CLSM for any type of intervention on social networks. As discussed above, we are able to recover relatively small treatment effects even with a small number of networks and individuals per network.

Since this is new methodology, there are several limitations. First, we present only one of several possible types of social network models for partially exchangeable networks. We are working on a general framework that extends all common statistical network models to causal and multiple network settings. Second, there is little work in goodness of fit diagnostics for latent space models in general. We have considered several measure of goodness of fit for our model but these are still in development. Finally, parameter estimation is somewhat time-consuming. Our MCMC algorithm is currently coded in R R Development Core Team (2011) and the run time is currently on the order of 10 minutes for 20 networks of size 10 and 10,000 chains. We plan to recode our MCMC algorithm in the C programming language to greatly increase processing speed.
Appendix A

References


Figure 1: MCMC Posterior Densities for Different Values of the Treatment Effect ($\alpha$)
Table 1: MCMC Posterior Modes and 95% Credible Intervals for Different Values of the Treatment Effect

<table>
<thead>
<tr>
<th>True $\alpha$</th>
<th>Posterior Mode</th>
<th>95% Credible Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2.81</td>
<td>(2.15, 3.47)</td>
</tr>
<tr>
<td>2</td>
<td>1.64</td>
<td>(0.79, 2.37)</td>
</tr>
<tr>
<td>1</td>
<td>0.81</td>
<td>(0.32, 1.51)</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.06</td>
<td>(-0.68, 0.61)</td>
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