Title:
Using Worked Examples Assignments in Classroom Instruction

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Abstract Body
Limit 4 pages single-spaced.

Background / Context:
Description of prior research and its intellectual context.

As highlighted by the National Math Panel Report (2008), consistent results from laboratory studies have demonstrated that interleaving worked examples with problems to solve improves learning for novices. The seminal work by Sweller and Cooper (1985) reported that learners improved more if they spent half their time solving problems and the other half studying examples of how similar problems were solved. Since then, over 35 articles have been published which replicate and extend those findings.

Despite the wealth of positive results, there are a number of factors pointing to the need for further work. For example, there is some counterevidence that it doesn’t always work or that certain constraints must be met. Renkl (1997) demonstrated that providing examples to students isn’t enough as students must also actively engage with the examples. Attention to the appropriate key information is also important (Ross, 1989). Further, studies don’t demonstrate uniform improvement, students with lower prior knowledge are thought to benefit more than those with higher knowledge (Kayluga, Chandler, & Sweller, 2001). In addition, nearly all studies have been conducted in laboratory settings and those conducted in classrooms were not integrated into regular classroom instruction (e.g., Ward & Sweller, 1990). A critical gap in the literature, then, is ecologically valid studies conducted in real world classrooms. This work can only be carried out using classroom friendly materials that instantiate principles derived from the worked example literature. A similar gap occurs between research and practice on this topic - our review of homework assignments given by teachers in 5 school districts showed that 95% of items were problems to solve. Out of the 128 total items, only 6 requested explanations, and only 1 included a worked example.

Purpose / Objective / Research Question / Focus of Study:
Description of the focus of the research.

The present effort fills both the research and practice gap. The purpose of the work is to create materials and tests that can be used flexibly in classrooms and which employ worked examples interleaved with practice problems for students to solve. A total of 42 assignments addressing some pre-Algebra and 8 major Algebra topics have been developed through an iterative process. A combination of single unit, double-unit, and year-long studies have been undertaken. Driven by the interests of the participating districts as well as a deep understanding of the literature, the effects of AlgebraByExample on learning has been explored with particular attention to the role of individual differences.

Setting and Population/Participants/Subjects:
Description of the research location. Description of the participants in the study: who, how many, key features, or characteristics.

The participating districts are all members of the Minority Student Achievement Network (MSAN) which is a self-formed alliance dedicated to addressing the achievement gap. Like most
MSAN districts, the 8 that have participated in this work are mostly located in inner-ring suburban areas (Ann Arbor (MI), Arlington (VA), Chapel Hill – Carrboro City (NC), Evanston Township (IL), Evanston / Skokie 65 (IL), Green Bay (WI), Shaker Heights (OH), the one exception is Madison Metropolitan School District (WI) which is urban). Although this paper will not cover all of the individual studies, the body of work as a whole has been conducted in 300 classrooms with over 6000 students participating. Students are in 7th-10th grades, with the majority in 8th or 9th enrolled in “regular” Algebra. The partnering districts have diverse student populations. In the study reported on here, 395 students are in the final sample, (204 Experimental, 191 Control; 53% male; 41% low SES) across 28 classrooms (16 teachers). The ethnicity breakdown was: 33% Caucasian, 40% African American, 15% Latino, 6% Asian, and 6% biracial. The average classroom was comprised of 64% minority students (African American, Latino, and biracial).

**Intervention / Program / Practice:**
*Description of the intervention, program, or practice, including details of administration and duration.*

Typically, textbooks and worksheets provide students with problems to solve in order to practice what they’ve been taught. Like typical materials, AlgebraByExample relies on teachers to teach and requires that students practice on their own. However, the innovative research-based and reality-checked design provides additional scaffolding. AlgebraByExample assignment items are grouped in pairs that include an “example” item for students to study and explain and a “your turn” item to solve. See Figure 1 for sample items.

------------------- Insert Figure 1 about here -------------------

As shown, an example presents the work of a fictional student whose name signals ethnicity and/or gender. The presented work features either a correct or incorrect attempt indicated with a check or “X” mark and phrases such as “Natasha solved this problem correctly” or “Natasha didn’t complete the graph correctly.”

Each example is followed by one or more questions that highlight key mathematical concepts students often misunderstand even after significant instructional time. Responding gives students practice analyzing, critiquing, and articulating mathematical arguments. Typically, students completed assignments in 10-20 minutes. Assignments were administered in class at the time deemed by the teacher to be most advantageous in relation to instruction on the relevant content area.

**Research Design:**
*Description of the research design.*

Random assignment was conducted at the level of the individual student. Approximately half of the students in each participating class were randomly assigned to the example-based group, in which they received the example-based assignments designed for the study. The other half was randomly assigned to the control group, in which they received an alternate version of the assignments that contained the same types of problems, but no examples or self-explanation prompts.
Each student received four example-based or four control assignments during each content unit; each student participated in two content units for a total of 8 assignments. The content units across the different classrooms were: Linear Equations, Pre-Algebra, Graphing, and Quadratics. Students completed a pre-test and post-test consisting of identical items that included both conceptual and procedural problems. To assess students’ procedural knowledge, we used 4 items that required students to carry out procedures to solve problems; all four problems were isomorphic to problems in the assignments, which were representative of the types of problems found in Algebra I textbooks and taught in Algebra I courses. To assess students’ conceptual knowledge, we used 22 items that measured understanding of crucial concepts from the four assignments (e.g., the meaning of the equals sign, the significance of negatives in terms, identification of like terms, etc.; Booth & Koedinger, 2008; Kieran, 1981; Knuth et al., 2006; Vlassis, 2004). Sample conceptual and procedural items can be found in Figure 2. Tests and assignments were paper-and-pencil documents.

Data Collection and Analysis:
Description of the methods for collecting and analyzing data.

All studies were conducted in a typical course setting, with all assignments and testing done as part of normal classroom activities. Hard copies of the assignments and assessments were collected, coded for completion, and in the case of the assessments, for accuracy as well. Mean pretest and posttest scores can be found in Table 1. All analyses were conducted with Hierarchical Linear Modeling (HLM) software.

To determine whether example-based assignments improve learning for algebra students, and whether there are differences in benefit based on either individual minority status or class composition, we computed two pairs of two-level hierarchical linear models with individual students nested in classrooms. The first pair of models (1_conc and 1_proc) included 5 predictors: pretest conceptual knowledge, pretest procedural knowledge, minority status, condition, and the interaction between condition and minority status. The second pair of models (2_conc and 2_proc) also included 5 predictors: pretest conceptual knowledge, pretest procedural knowledge, percentage of minority students in the classroom, condition, and the interaction between condition and the percentage of minority students in the classroom. Any results not reported failed to reach significance.

Findings / Results:
Description of the main findings with specific details.

Conceptual Knowledge. In both models, posttest conceptual scores were positively associated with pretest conceptual scores (βs=.25 and .27, Fs(26, 389) = 5.53 and 5.92, both ps < .001). In model 1_conc, posttest conceptual scores were inversely associated with the individual student’s minority status (β=-.05, F(26, 389) = -2.25, p = .003), and in model 2_conc they were inversely associated with the percentage of minority students in the classroom (β=-.22, F(26, 389) = -3.80, p < .001). This indicates that having a higher conceptual score at pretest and either being a non-minority student or being in a class with a lower percentage of minority students.
predicts higher posttest scores. No interactions between condition and either type of minority status information were found.

Procedural knowledge. In both models, posttest procedural scores were positively associated with pretest procedural scores ($\beta$= .23 and .24, $F$s(26, 389) = 3.37 and 3.52, both $p$s < .001) and pretest conceptual scores ($\beta$s= .24 and .27, $F$s(26, 389) = 3.02 and 3.43, $p$s = .003 and .001). In model $1_{\text{proc}}$, posttest procedural scores were inversely associated with the individual student’s minority status ($\beta$= -.10, $F$(26, 389) = -2.76, $p$ = .007), and in model $2_{\text{proc}}$ they were inversely associated with the percentage of minority students in the classroom ($\beta$= -.27, $F$(26, 389) = -2.50, $p$ = .02). This indicates that having a higher procedural score at pretest, having a higher conceptual score at pretest, and either being a non-minority student or being in a class with a lower percentage of minority students predicts higher posttest scores. In model $1_{\text{proc}}$, there was no interaction between condition and individual minority status. However, in model $2_{\text{proc}}$ there was a trend toward a positive association with the interaction between the slope of Condition and the percentage of minority students in the classroom ($\beta$= .17, $F$(26, 389) = 1.75, $p$ = .08). Follow-up examination of simple slopes indicated that there was an effect of minority percentage in the control group (higher minority classes improved less than lower minority classes; $t$ = -2.50, $p$ = .01), but in the example-based group, students in both high and low-minority classes learned equally ($t$ = -0.68, $p$ = .50). Put another way, there were trends toward a main effect of condition in higher-minority classrooms (at and above 50% minority, $t$=1.83, $p$=0.07). This indicates that for students in high-minority classrooms, being in the example-based condition predicted higher posttest scores.

Conclusions:
*Description of conclusions, recommendations, and limitations based on findings.*

Results demonstrate that students in high-minority populations benefit more from example-based assignments than do those in lower-minority groups in terms of an increase in their procedural performance. Results also indicate that conceptual learning occurs more readily for students in low-minority classrooms – students in higher-minority classrooms do not tend to learn as much. However, for students in high-minority classrooms, being in the example-based condition leads to greater learning than the control condition. Thus, providing correct and incorrect worked examples can improve learning, perhaps especially for students who are at a disadvantage.
Appendices
Not included in page count.

Appendix A. References
References are to be in APA version 6 format.


Appendix B. Tables and Figures

Not included in page count.

Figure 1: Excerpts from the example-based and control versions of the assignment on Solving Multi-Step Equations

<table>
<thead>
<tr>
<th>Example-based</th>
<th>Control</th>
</tr>
</thead>
</table>
| 3. \[ \frac{3}{4x - 6} = 5 \]  
**Correct Example**  
Sasha solved this problem correctly. Here are the steps she used to solve the problem:  
- \[ 3(4x - 6) = 5 \cdot (4x - 6) \]  
- \[ 3 - 6(2x - 3) \]  
- \[ 3 - 12x + 18 \]  
- \[ 21 = 12x \]  
- \[ \frac{21}{12} = x \]  
Why did Sasha multiply both sides by \((4x - 6)\) in the highlighted step? | 4. \[ \frac{3}{6 - 4x} = 5 \]  
**Incorrect Example**  
Umih tried to solve this problem, but she didn't do it correctly. Here is her first step to solve the problem:  
- \[ 3x = 4x - 6 + 3 \]  
- \[ 3x = 4x - 3 \]  
What was Umih's first step to solve the problem?  
Why didn't Umih's first step keep the equation in balance? |
| 5. \[ 3x = 4x - 6 + 5 \]  
**Incorrect Example**  
Umih tried to solve this problem, but she didn't do it correctly. Here is her first step to solve the problem:  
- \[ 3x = 4x - 6 + 5 \]  
- \[ 3x = 4x - 1 \]  
What was Umih's first step to solve the problem?  
Why didn't Umih's first step keep the equation in balance? | 6. \[ 3 = 4 + 6x - 5x \]  
**Incorrect Example**  
Umih tried to solve this problem, but she didn't do it correctly. Here is her first step to solve the problem:  
- \[ 3 = 4 + 6x - 5x \]  
- \[ 3 = 4 + x \]  
What was Umih's first step to solve the problem?  
Why didn't Umih's first step keep the equation in balance? |
| 3. \[ \frac{5}{2f - 4} = 9 \] | 4. \[ \frac{4}{8 - 7r} = 5 \] |
| 5. \[ 2k = 5k - 9 + 6 \] | 6. \[ 8 = 7 + 4v - 3v \] |
### Figure 2: Sample assessment items for Experiment 2

<table>
<thead>
<tr>
<th>Pre-Algebra</th>
<th>Conceptual</th>
<th>Procedural</th>
</tr>
</thead>
<tbody>
<tr>
<td>State whether each of the following is equivalent to ( x + 4 - 2 + x ):</td>
<td></td>
<td>Find the quotient for the expression and write in simplest form. Show all of your work:</td>
</tr>
<tr>
<td>a. ( (x + 4) - (2 + x) )</td>
<td>Yes</td>
<td>( \frac{4}{5} \div -2 )</td>
</tr>
<tr>
<td>b. ( 4 + x - 2 + x )</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>c. ( x + (4 - 2) + x )</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>d. ( x + 4 - x + 2 )</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>e. ( (x + 4) + (-2 + x) )</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>f. ( x + 4 + x - 2 )</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>g. ( x + 2(2 - 1) + x )</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Graphing</th>
<th>Conceptual</th>
<th>Procedural</th>
</tr>
</thead>
<tbody>
<tr>
<td>State whether each of the following is true for the line: ( (y - 3) = 2(x + 1) ):</td>
<td></td>
<td>Find the ( x )- and ( y )-intercepts. Then use them to graph the equation:</td>
</tr>
<tr>
<td>a. The line goes through ((1, -3))</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>b. The slope-intercept form of the line is ( y = 2x + 5 )</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>c. The line has a slope of 2</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>d. The line goes through ((3, -1))</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>e. The line has a slope of ( \frac{1}{2} )</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>f. The line goes through ((-1, 3))</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>g. The slope-intercept form of the line is ( y = 2x - 1 )</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Quadratics</th>
<th>Conceptual</th>
<th>Procedural</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identify the type of each function:</td>
<td></td>
<td>Solve for ( z ) using the quadratic formula. Show all of your work.</td>
</tr>
<tr>
<td>a. ( y = x^2 + 6 )</td>
<td>Linear</td>
<td>Quadratic</td>
</tr>
</tbody>
</table>
| b. \[
\begin{array}{c|cccc}
 x & 2 & 4 & 6 & 8 \\
 y & 1 & 4 & 7 & 10 \\
\end{array}
\] | Linear | Quadratic | Exponential |
| c. \( y = 2^x + 6 \) | Linear | Quadratic | Exponential |
| d. \[
\begin{array}{c|cccc}
 x & 3 & 4 & 5 & 6 \\
 y & 11 & 18 & 27 & 38 \\
\end{array}
\] | Linear | Quadratic | Exponential |
| e. \( y = 2x + 6 \) | Linear | Quadratic | Exponential |
Table 1

**Experiment 2: Means and Standard Deviations for Conceptual and Procedural Scores at Pretest and Posttest by Condition and Minority Status**

<table>
<thead>
<tr>
<th>Condition</th>
<th>MAP  Pre</th>
<th>MAP  Pos</th>
<th>PAP  Pre</th>
<th>PAP  Pos</th>
<th>PAV  Pre</th>
<th>PAV  Pos</th>
<th>ENJ  Pre</th>
<th>ENJ  Pos</th>
<th>PC   Pre</th>
<th>PC   Pos</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td>.19 (.23)</td>
<td>.43 (.26)</td>
<td>.64 (.17)</td>
<td>.75 (.16)</td>
<td>5.1 (1.4)</td>
<td>5.1 (1.4)</td>
<td>4.6 (1.7)</td>
<td>4.6 (1.7)</td>
<td>4.5 (1.7)</td>
<td>4.5 (1.7)</td>
</tr>
<tr>
<td>Control</td>
<td>.18 (.21)</td>
<td>.42 (.29)</td>
<td>.64 (.18)</td>
<td>.75 (.17)</td>
<td>5.0 (1.6)</td>
<td>5.1 (1.3)</td>
<td>4.3 (1.7)</td>
<td>4.5 (1.5)</td>
<td>4.2 (1.8)</td>
<td>4.2 (1.7)</td>
</tr>
</tbody>
</table>

*Note.* Mean(SD)