Title: Development of a Comprehensive Intervention to Improve Children’s Understanding of Math Equivalence

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Mathematical equivalence is the relation between two quantities that are interchangeable (Kieran, 1981), and its symbolic form (i.e., the equal sign) specifies that the two sides of an equation are equal and interchangeable. It is widely regarded as one of the most important concepts for developing young children’s algebraic thinking (e.g., Knuth, Stephens, McNeil, & Alibali, 2006). Unfortunately, most children in the U.S. struggle to understand this fundamental concept (Baroody & Ginsburg, 1983; Carpenter, Franke, & Levi, 2003). Children’s misconceptions are apparent when they are asked to solve “math equivalence problems,” which are equations with operations on both sides of the equal sign (e.g., 3 + 4 = 5 + _). Instead of solving such problems correctly (e.g., 2), they typically just add up all the numbers (e.g., 12), or add the numbers before the equal sign (e.g., 7).

McNeil and Alibali (2005b) advanced a “change-resistance” account of children’s difficulties with math equivalence, arguing that children’s difficulties are due to children’s overly narrow experience with arithmetic in school. In the U.S., children learn arithmetic in a procedural fashion for years before they learn to reason about equations relationally, as expressions of math equivalence. Moreover, arithmetic problems are typically presented with operations on the left side and the “answer” on the right (e.g., 3 + 4 = 7, Seo & Ginsburg, 2003), a format that does not highlight the interchangeable nature of the two sides of an equation. As a result of this narrow experience, children extract patterns that do not generalize beyond arithmetic. These patterns are referred to as “operational patterns” (McNeil & Alibali, 2005b) because they are derived from experience with arithmetic operations, and they reflect operational rather than relational thinking (cf. Jacobs, Franke, Carpenter, Levi, & Battey, 2007).

According to a “change-resistance” account, small modifications to arithmetic practice that reduce children’s reliance on these operational patterns may help children develop a better understanding of math equivalence. In support of this view, several studies have found that specific modifications to arithmetic practice help children construct a better understanding of math equivalence. For example, in a randomized experiment, children who practiced problems in nontraditional formats (e.g., __ = 8 + 4) constructed a better understanding of math equivalence than children who practiced problems in a traditional format (e.g., 8 + 4 = __) or did not receive any extra practice (McNeil, Fyfe, Petersen, Dunwiddie, & Brletic-Shipley, 2011).

In addition to modified arithmetic practice, three other interventions have been found to improve children’s understanding of math equivalence. The first involves presenting the equal sign outside of arithmetic contexts (e.g., 28 = 28, 1 foot = 12 inches) (McNeil, 2008). The second involves “concreteness fading” exercises, which help to strengthen the mappings between concrete, real-world relational contexts (e.g., sharing, balancing a scale) and the corresponding mathematical symbols (e.g., Arabic numerals, operators, the equal sign) (Fyfe & McNeil, 2009). The third involves activities that require children to compare and explain different problem formats and problem-solving strategies (cf. Hattikudur & Alibali, 2010; Rittle-Johnson & Star, 2007). Each of these component interventions has been shown to help children improve their understanding of math equivalence; however, none has been able to get children to a “mastery-level” understanding (defined below).
Purpose / Objective / Research Question / Focus of Study:
Description of the focus of the research.

Our goal is to develop and evaluate a comprehensive intervention that helps second grade children achieve a mastery-level understanding of math equivalence. Children who achieve mastery should exhibit at least 75% accuracy when solving and encoding math equivalence problems, and they should define the equal sign relationally.

Setting:
Description of the research location.

During Year 1, we conducted one-on-one sessions with children at a Boys & Girls Club in a mid-sized urban area in the Midwestern U.S. During Year 2, our intervention was used in a classroom in a public elementary school in a large city in the Southeastern U.S.

Population / Participants / Subjects:
Description of the participants in the study: who, how many, key features, or characteristics.

During Year 1, 26 children participated; however, three children moved away or stopped coming to the center for unknown reasons. Thus, the final sample for Year 1 included 23 2nd-grade children (ages 7-8; 13 boys, 10 girls; 70% African American or black, 17% white, 9% other, and 4% Hispanic or Latino). During Year 2, 23 2nd-grade children participated (ages 7-8; 10 boys, 13 girls). The racial/ethnic makeup of the participating school was 83% white, 13% African American or black, 2% Asian, and 2% Hispanic or Latino.

Intervention / Program / Practice:
Description of the intervention, program, or practice, including details of administration and duration.

Our comprehensive intervention is designed to reduce children’s reliance on the operational patterns and promote relational thinking. It draws on and extends the four component interventions shown to improve children’s understanding of math equivalence (described above). Specifically, throughout our intervention, we are including modified arithmetic practice that involves nontraditional problem formats (e.g., __ = 8 + 4), problems grouped by equivalent values (e.g., 5 + 2, 3 + 4, 1 + 6), and relational phrases such as “is equal to” in place of the equal sign. In addition, the early sessions focus on lessons that introduce the equal sign outside of arithmetic contexts (e.g., 7 = 7), the middle sessions focus on “concreteness fading” exercises, and the later sessions focus on activities that require children to compare and explain different problem formats (e.g., 7 = 5 + 2 compared to 5 + 2 = 7) and different problem-solving strategies.

Research Design:
Description of the research design.

In Year 1, we conducted a one-on-one design experiment in which we ran a series of individual tutoring sessions. We compared and contrasted the learning trajectories of children who received all four components of the intervention to those of children who received alternative versions with only three components (e.g., no practice with non-traditional formats; no practice with concreteness fading). Results suggested that the full version was best. Thus, in
Year 2 we used that “ideal” version to create a teacher manual and workbook set for use in a classroom-based design experiment. After this iterative development process, we will perform a randomized, classroom-based pilot study in Year 3 (during the 2013-2014 school year) to further demonstrate the effectiveness and feasibility of the comprehensive intervention.

Data Collection and Analysis:
Description of the methods for collecting and analyzing data.

During Year 1, children first completed a pretest to assess their understanding of math equivalence. It included three components: (a) solving math equivalence problems, (b) encoding math equivalence problems, and (c) defining the equal sign. This three-component measure of understanding of math equivalence has exhibited high levels of reliability and validity in previous work (e.g., Knuth et al., 2006; McNeil & Alibali, 2005a, b). To assess solving, children solved a set of math equivalence problems (e.g., $1 + 5 = \_ + 2$) and explained their solutions. To assess encoding, children reconstructed math equivalence problems after viewing each for five seconds. To assess defining the equal sign, the experimenter pointed to an equal sign and asked children: (1) “What is the name of this symbol?” (2) “What do you think this symbol means?” and (3) “Can it mean anything else?” After completing the pretest, children received 34 15-minute sessions spread over the course of about 10 weeks in which they worked one-on-one with trained tutors from our team. Finally, children completed a posttest that was similar to the pretest. We videotaped the sessions and tests, collected all children’s work, assessed children’s enjoyment, asked tutors to complete log sheets after every session, and gave tutors a brief retrospective survey at the end of each wave. We used all of this data to make improvements to the intervention after each wave. The end product of Year 1 was a sequence of 32 15-minute sessions for use in the classroom-based design experiment.

During Year 2 (the current year), we have been continuing to improve the comprehensive intervention. Our teacher collaborator (Brletic-Shipley) used the intervention in her classroom with the goal of optimizing the intervention for use in a classroom setting. She provided constant feedback about usability and effectiveness, offered suggestions for improvements, and tried out agreed-upon improvements throughout the year. Her students completed a pretest, mid-test, and posttest similar to those given to children in the one-on-one design experiment.

Decades of research have already shown that most children (ages 7-11) in the U.S. do not demonstrate even a basic understanding of math equivalence under “business as usual” conditions. Our research team conducted many of these studies, so we have ample data on children’s performance on the specific measures we are using in the present study. For example, of the children who participated in our previous control conditions ($N = 235$), only 20% solved any math equivalence problem correctly, only 49% encoded any math equivalence problem correctly, and only 6% defined the equal sign correctly. These numbers are consistent with other studies. We are using these data as the “worst-case” benchmarks (effects expected with no treatment) for comparison to the performance of children who receive our comprehensive intervention. In addition, we also have ample data on children’s performance after various interventions. Although many previous interventions have been successful in terms of improving children’s understanding of math equivalence, none have produced anything close to mastery-level understanding. For example, children who received McNeil et al.’s (2011) “nontraditional arithmetic practice” intervention not only outperformed the worst-case benchmarks, but also outperformed children who were randomly assigned to a traditional arithmetic practice intervention. However, their performance was not even close to mastery. Only 20% solved at
least 75% of the math equivalence problems correctly, only 22% encoded at least 75% of the problems correctly, and only 15% defined the equal sign correctly. We are using these data as the “best-case” benchmarks (effects expected with a successful component intervention) for comparison to the performance of children who receive our comprehensive intervention.

Findings / Results:
Description of the main findings with specific details.

Children from Year 1 were from disadvantaged backgrounds, and they did not demonstrate an understanding of math equivalence on any of the pretest measures. Many struggled with basic arithmetic. However, all 23 children improved their arithmetic skill and changed their thinking about math equivalence as a result of participating (see Figures 1, 2, and 3). Children not only changed their thinking, but also constructed correct ways of thinking. After the intervention, 78% of children defined the equal sign as a relational symbol of equivalence; 83% solved at least one math equivalence problem correctly; and 52% reconstructed at least one problem correctly. These numbers are impressive, especially considering the fact that in previous studies with children from more advantaged backgrounds only 6% of children defined the equal sign correctly, only 20% solved at least one math equivalence problem correctly, and only 49% reconstructed at least one problem correctly (see worst-case benchmarks above).

In Year 2, we found that children’s understanding math equivalence improved over the course of the intervention (see Figures 4, 5, and 6). Prior to beginning the intervention, only 43% of students could solve one or more math equivalence problems correctly. At mid-test, 100% of students could. Even more impressive is that 70% of these students demonstrated what we consider “mastery-level” performance by solving at least 75% of the math equivalence problems correctly. Additionally, 39% of the students encoded the structure of at least 75% of problems correctly and 61% defined the equal sign relationally. At posttest, students showed even greater improvements. Specifically, 96% of the students solved at least 75% of the problems correctly, 70% encoded the structure of at least 75% of problems correctly, and 70% defined the equal sign relationally. Thus, our “best-case” benchmarks were surpassed in each task.

Conclusions:
Description of conclusions, recommendations, and limitations based on findings.

Despite the progress the field has made over the past few decades in terms of understanding the nature of children’s difficulties with math equivalence, this theoretical progress has not been translated into large-scale pragmatic changes in the ways children are taught. To date, researchers have yet to develop a comprehensive intervention that produces mastery-level understanding of math equivalence in most elementary school children. If we expect schools to make large-scale changes, then we need to provide them with a comprehensive intervention that is effective, affordable, portable, and easy to administer.

Our findings from both the one-on-one tutoring sessions in Year 1 and the classroom setting in Year 2 provide us with evidence to suggest that our comprehensive intervention leads children to construct an understanding of math equivalence that surpasses the worst-case and best-case benchmarks seen in previous studies. However, our results to date are only from the iterative design phase of our project, so our ability to make causal claims is limited. In the upcoming year, we will further test the effectiveness and feasibility of the intervention in a randomized, classroom-based pilot study.
Appendices
Not included in page count.

Appendix A. References
References are to be in APA version 6 format.


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Appendix B. Tables and Figures
Not included in page count.

Figure 1

![Year 1: Equation Solving at Pretest and Posttest](image1)

Figure 2

![Year 1: Encoding Performance at Pretest and Posttest](image2)
Figure 3

**Year 1: Defining the Equal Sign at Pretest and Posttest**

![Bar chart showing the percentage of children understanding the equal sign at pretest and posttest. The categories are Other, Operational, and Relational.]

Figure 4

**Year 2: Equation Solving at Pretest and Posttest**

![Bar chart showing the percentage of children solving equations at pretest and posttest. The categories are at least 1 correct and at least 75% correct.]

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Figure 5

Year 2: Encoding Performance at Pretest and Posttest

% of children

At least 1 correct
At least 75% correct

Pretest
Posttest

Figure 6

Year 2: Defining the Equal Sign at Pretest and Posttest

% of children

Other
Operational
Relational

Pretest
Posttest