Evaluating the Efficacy of a Learning Trajectory for Early Shape Composition

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Background

The use of learning trajectories (LTs) in early mathematics instruction has received increasing attention from policy makers, educators, curriculum developers, and researchers (Baroody, Clements, & Sarama, in press; Clements & Sarama, 2014; 2011; Maloney, Confrey, & Nguyen, 2014; Sarama & Clements, 2009) and are generally deemed as a useful tool for guiding standards, instructional planning, and assessment (Frye et al., 2013; NRC, 2009). Despite these recommendations, little research has directly tested the specific contributions of LTs to teaching compared instruction provided without LTs (Frye et al., 2013). The primary goal in the present study was to compare the learning of preschool children who received instruction on shape composition based on an empirically-validated LT to those who received an equal amount of instruction not following the developmental order of an LT. Only then *should* education research on learning trajectories inform practice.

Purpose / Objective / Research Question

This study tested one defining attribute of LTs, asking the following research question: Does instruction in which LT levels are taught consecutively (e.g., for children at level n, instructional tasks from level $n + 1$, then $n + 2$, consistent with Piaget's moderate novelty principle) result in greater learning than instruction that immediately and solely targets level $n + 2$ (the "skip-levels" approach)? We also investigated whether entering knowledge or ethnic group were significant moderators of differences. We measured children's competence in two ways, scoring correctness only, or including strategy sophistication levels as well.

Setting and Participants

Participants were pre-kindergartners enrolled in a large public-school district with a diverse population of 22,861 elementary school children, including 1% American Indian, 8% Asian, 11% Black, 19% Hispanic, 1% Pacific Islander, 55% white (non-Hispanic), and 5% 2 or more ethnic groups; also, 28% are free/reduced lunch and the mobility rate is 17%.

Interventions

We developed an elaborated, scripted instructional unit on shape composition following the LT (Fig. 1). Instruction was straightforward: Children were invited to solve puzzles. Several puzzles at the appropriate level were offered to promote child choice and maintain interest. The LT group was offered puzzles and provided instruction at the level directly following the level at which they had evinced competence (adjusted dynamically); e.g., if a child could not solve a problem from a newly-introduced level, the interventionist might draw one internal line as a scaffold. The skip-levels group was given puzzles at the goal level (Shape Composer), with encouragement for effort but without scaffolding that would reduce the level of the task.

Research Design

Procedures

We trained the interventionists to deliver the activities. Interventionists piloted these activities and video recordings of their instruction were reviewed by the authors using the fidelity measure, with feedback given to interventionists individually.

We pretested all children for whom we had obtained consent and examined the resulting data to determine initial instructional level, then randomly assigned them to small groups, and then the 2 or 4 groups in each classroom were randomly assigned to condition. This design provides control for variance due to community, school, and teachers. We thus implemented a three-level randomized block design with fixed effects.

Interventionists then implemented the treatments. The authors checked the fidelity of instruction on 10% of the lessons for each interventionist using the fidelity measure and offered

feedback for "fine-tuning" instruction. We successfully implemented 6-10 days (M = 8.14) for the shape composition instructional unit lasting an average of 8.59 minutes each session (including introduction, activities, transitions, etc.). After the instructional period was completed, we posttested all children remaining in the study at the end of the instruction.

We used a Cluster Randomized Trial (CRT) design, with children embedded within groups (which are embedded within classrooms).

Results

This first set of analyses was conducted using the Rasch scores based only on correctness (ignoring process behaviors). Means and standard deviations by group are presented in Table 1. The unconditional model indicated that about 14% of the variance in posttest scores lay between groups ($\sigma^2 = 2.39$, $\tau = .40$, $\rho = .143$). In the final model, the intervention had significant effects on children's mathematics performance (b = .64, SE = .25, $p = .013$, g = .39). Child-level moderators included intervention condition by each of the key pretest variables (pretest score, age, gender, race, and number of intervention sessions). The group-level moderator was the interaction of intervention condition by group pretest. Of these child- and group-level interaction terms, none were significant when entered into the model.

This second, similar, set of analyses was conducted using the Rasch scores that incorporated solution processes as well as correctness. Results were similar; however, the intervention was most effective for children who started the intervention with lower scores of the pretest (see Fig. 3). We computed means for all items and categorized according to levels of transfer (Table 6).

Discussion

Although instruction was brief, we found that LT instruction was more effective than skip-levels instruction. Examination of individual items confirmed that the LT group made more progress toward completely correct solutions to the assessment items and toward using processes at higher levels of sophistication. These effects were especially pronounced on tasks similar to the target level. This is notable, as the goal level of thinking was achieved more frequently by children who experienced fewer tasks at that level, the LT group. However, the benefits of the LT treatment did not extend to medium or far transfer. Perhaps the cognitive challenge that the target-level tasks presented to the children in the skip-levels group, stimulating them to use spatial imagery.

None of the moderators were significant except that the intervention, when measured using the scoring system that incorporated strategy use, was most effective for children who started the intervention with lower scores of the pretest.

Several caveats should be noted. Instruction was provided by trained interventionists to small groups, not teachers to full classes. Results are limited to one domain of mathematics; future research must involve other domains, as it is possible that the more effective method of instruction varies by topic.

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Table 1. Means and standard deviations of Rasch scores on correctness

	Learning Trajectories (n = 81)		Skip Level Condition (n = 64)	
Condition	M	SD	M	SD
Pretest	-3.06	1.48	-3.27	1.86
Posttest	-2.91	1.48	-3.59	1.83

Table 2. Effects of the learning trajectories intervention on correctness

	Coefficient	SE	p
Intercept	-3.40	0.17	<.001
Level 1 (child)			
Pretest	0.07	0.14	.523
Age	0.45	0.22	.048
Gender	0.50	0.25	.047
Race	-0.43	0.35	.225
Number of sessions	0.01	0.14	.875
Level 2 (group)			
Group Pretest	0.36	0.16	.029
Condition	0.64	0.25	.013

Table 3. Means and standard deviations of Rasch scores on correctness and use of processes

	Learning Trajectories (n = 81)		Skip Level Condition (n = 64)	
Condition	M	SD	M	SD
Pretest	-1.63	1.79	-1.85	2.11
Posttest	-0.88	1.53	-1.73	2.25

Table 4. Effects of the learning trajectories intervention on correctness and use of processes

	Coefficient	SE	p
Intercept	-1.93	0.25	<.001
Level 1 (child)			
Pretest	0.34	0.11	.003
Age	0.28	0.26	.272
Gender	0.50	0.28	.076
Race	0.18	0.40	.662
Intervention sessions	0.01	0.09	.941
Level 2 (group)			
Group Pretest	0.51	0.10	<.001
Condition	0.75	0.29	.012

Table 5. Effects of the learning trajectories intervention on correctness and use of processes including interactions

	Coefficient	SE	p
Intercept	-1.91	0.30	<.001
Level 1 (child)			
Pretest	0.64	0.15	<.001
Age	0.68	0.38	.076
Gender	0.15	0.43	.731
Race	1.09	0.67	.109
Intervention sessions	0.01	0.12	.915
Pretext x condition	-0.57	0.21	.010
Age x condition	-0.55	0.52	.293
Gender x condition	0.60	0.57	.293
Race x condition	-1.35	0.34	.111
Intervention sessions x condition	0.00	0.19	.988
Level 2 (group)			
Group Pretest	0.50	0.14	<.001
Condition	0.69	0.41	.095
Group Pretest x condition	0.02	0.21	.900

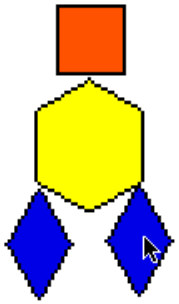
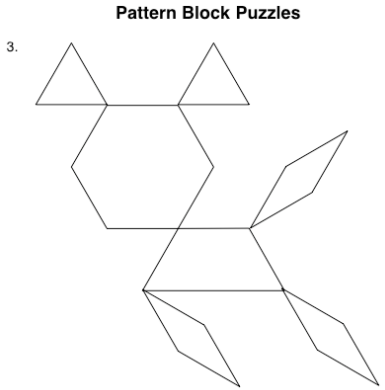
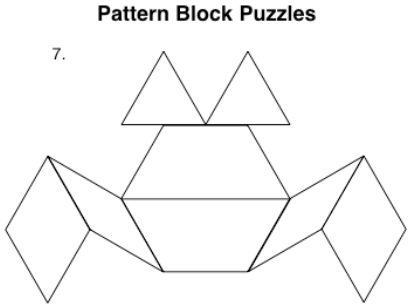
Table 6. Means and standard deviations of correct (A codes) and process (B, C, D codes) for items categorized by level of transfer.

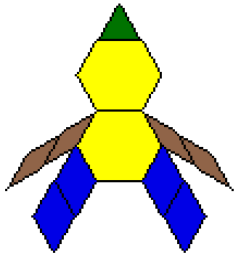
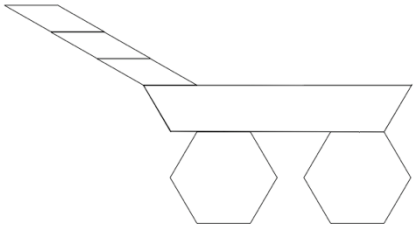
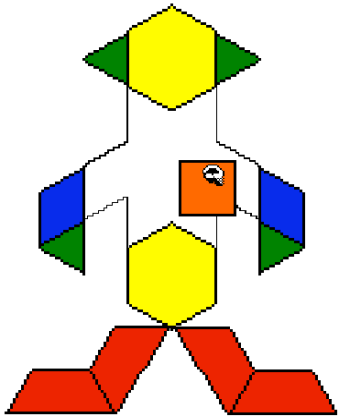
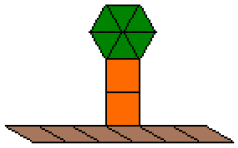
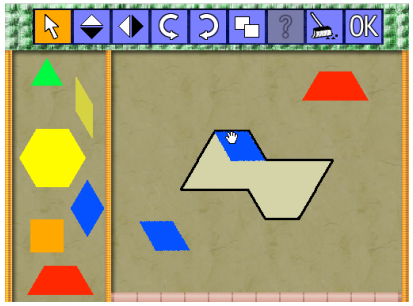
		Learning Trajectory Condition				Skip Level Condition			
		Pretest		Posttest		Pretest		Posttest	
Item #	Descriptor of item	Mean	SD	Mean	SD	Mean	SD	Mean	SD
		Near Transfer							
1A	See Fig. 2.	0.95	0.61	1.51	0.61	0.89	0.59	1.20	0.72
1B		1.03	0.74	1.64	0.75	0.95	0.75	1.47	0.94
1C		0.97	0.64	1.43	0.48	0.95	0.67	1.34	0.73
1D		1.09	0.72	1.55	0.59	1.00	0.71	1.41	0.74
2A	Similar to Item 1.	0.53	0.55	0.88	0.70	0.53	0.56	0.76	0.67
2B		0.57	0.58	1.15	0.87	0.57	0.63	0.91	0.90
2C		0.59	0.60	0.97	0.65	0.55	0.60	0.86	0.81
2D		0.68	0.74	1.15	0.80	0.65	0.74	0.91	0.71
3A	Similar to Item 1.	0.67	0.50	0.95	0.55	0.71	0.46	0.80	0.61
3B		0.42	0.58	0.88	0.77	0.49	0.51	0.74	0.83
3C		0.44	0.60	0.83	0.67	0.53	0.58	0.66	0.71
3D		0.49	0.71	0.90	0.81	0.57	0.71	0.69	0.79
		Medium Transfer							
4	Fill in identical puzzles in different ways.	0.00	0.00	0.05	0.22	0.00	0.00	0.00	0.00
5A	Use 4 of 6 shapes to fill puzzle.	0.22	0.42	0.33	0.47	0.17	0.38	0.26	0.44

5B		0.36	0.49	0.56	0.50	0.37	0.50	0.67	0.48
6A	How many of one shape will fill another.	0.10	0.30	0.19	0.39	0.16	0.37	0.18	0.39
6B		0.65	0.88	1.64	0.73	0.83	0.89	1.23	0.93
7	How many of which shapes needed to fill puzzle.	--	--	0.00	0.00	--	--	0.02	0.13
8	How many of which shapes needed to fill puzzle.	--	--	0.20	0.40	--	--	0.10	0.30
				Far Transfer					
9	Choose shape created by composing shapes.	0.34	0.48	0.33	0.48	0.29	0.46	0.39	0.49
10	Choose shapes created by decomposing shape.	0.18	0.38	0.19	0.39	0.17	0.38	0.18	0.39

Figure 1

Relevant Levels from the Learning Trajectory for Composition of Geometric Shapes (adopted from Clements & Sarama, 2014; Sarama & Clements, 2009)

Developmental Progression	Example Behavior	Instructional Tasks
Piece Assembler Makes pictures in which each shape represents a unique role (e.g., one shape for each body part) and shapes touch. For this study, Target Level - 2	Make a picture  Solve a puzzle Fills simple puzzles such as those at the right using trial and error.	In the first "Pattern Block Puzzles" tasks, each shape is not only outlined, but touches other shapes only at a point, making the matching as easy as possible. Students merely match pattern blocks to the outlines.  Then, the puzzles moved to those that combine shapes by matching their sides, but still mainly serve separate roles. 
Picture Maker Puts several shapes together to make one part of a picture (e.g., two shapes for one arm). Uses trial and error and does not anticipate creation of new	Make a picture	The "Pattern Block Puzzles" at this level start with those where several shapes are combined to make one "part," but internal lines are still available.	Later puzzles in the sequence require combining shapes to fill one or more regions, without the guidance of internal line segments.

Developmental Progression	Example Behavior	Instructional Tasks
geometric shape. Chooses shapes using "general shape" or side length. For this study, Target Level - 1	 Solve a Puzzle Fills easy puzzles that suggest the placement of each shape (but note to the fart right that they student is trying to put a square in the puzzle where its right angles will not fit—this remains a levels of "trial and error" strategies).	 
Shape Composer. Composes shapes with anticipation ("I know what will fit!"). Chooses shapes using angles as well as side lengths. Rotation and flipping are used intentionally to select and place shapes. For this study, Target Level	Make a picture  Solve a Puzzle Solves puzzles using side and angle recognition and matching are correct	The "Pattern Block Puzzles" and "Piece Puzzler" activities have no internal guidelines and larger areas; therefore, students must compose shapes accurately. 

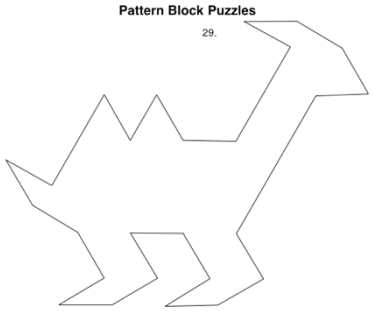
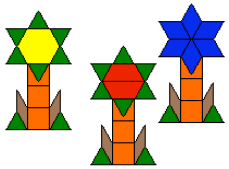
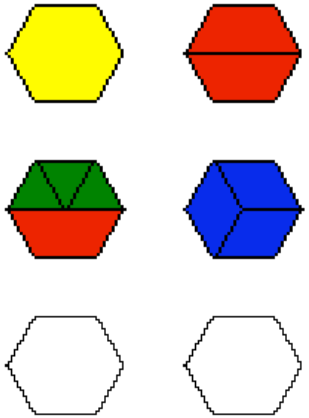
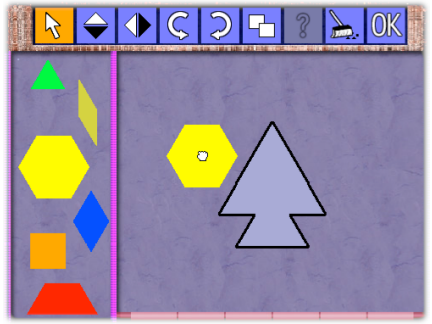
Developmental Progression	Example Behavior	Instructional	Tasks
			
Substitution Composer Makes new shapes out of smaller shapes and uses trial and error to substitute groups of shapes for other shapes to create new shapes in different ways. For this study, Target Level + 1	Make a picture with intentional substitutions 	At this level, students solve "Pattern Block Puzzles" in which they must substitute shapes to fill an outline in different ways. 	"Piece Puzzler" tasks are similar; the new task here is to solve the same puzzle in several different ways. 

Figure 2: Item 1 of the Shape Composition Test

Give the child the set of pattern blocks, randomly mixed in front of them, and the picture of a puzzle (right). Say: "**Use pattern blocks to fill this puzzle. Put them together with full sides touching."**

Code 1A (*Very small gaps or misalignments that can be attributed to fine motor limitations are acceptable*)

0 = incorrect (placed no shapes *or* placed shapes but not one "fit" the puzzle form, where *fit = at least one side aligned, with no "hangover" outside the puzzle.)*
1 = "partially correct" (one or more shapes "fit" but there were one or more gaps or "hangovers")
2 = correct (completed puzzle accurately; no gaps or "hangovers")
NR = no response

Code 1B *For all but 1-2 of the shapes,*

0 = selection of shapes not focused on completing puzzle (e.g., selects all red trapezoids)
1 = was hesitant or not systematic (e.g., used cycles of trial and error)
2 = completed the puzzle correctly, systematically, but may be "halting"
3 = completed the puzzle correctly, immediately, and confidently
9 = N/A (not applicable)

Code 1C *For all but 1-2 of the shapes,*

0 = selection of shapes not focused on completing puzzle (e.g., selects all red trapezoids)
1 = turned shapes after placing on puzzle in an attempt to get them to fit
2 = turned shapes into correct orientation prior to placing them on the puzzle
9 = N/A

Code 1D *For all but 1-2 of the shapes,*

0 = selection of shapes not focused on completing puzzle (e.g., selects all red trapezoids)
1 = tried out shapes by picking them seemingly at random, then putting them back if they did not look right, so seemingly trial and error
2 = appeared to search for "just the right shape" that they "know will fit" and then finding and placing it.
9 = N/A

Figure 3. Interaction between child-level pretest score and intervention condition predicting posttest scores that account for process usage.