Abstract Title Page

Title: Social Network Mediation Models

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Preferred Session:
Research Methods

Social Network Mediation Models

Background / Context

Social networks are particularly informative for studies aiming to change the ways in which individuals interact, whether it is an increase in collaboration among certain researchers, a push toward small learning communities of teachers or breaking up adolescent cliques. Social networks may provide insight into the mechanisms that affect individual outcomes and act as a powerful mediator between the intervention and outcome, especially in large-scale interventions.

Of course, the ultimate purpose or theory of action is not simply to change the social or professional relationships among the individuals in the network, it is rather to influence network-level or individual outcomes as a result of changes in the network. In educational studies, for example, changes in the social network are merely the mechanisms through which schools become more effective: teachers become better teachers and students become better learners. For example, an initiative aimed at in- creasing teacher collaboration does so in the hopes of improving teacher quality and student outcomes.

Purpose / Objective / Research Question / Focus of Study

The focus on this work is to introduce a general framework for modeling social networks as mediators. In addition, we will introduce the Hierarchical Mixed Membership Stochastic Blockmodel (HMMSB) for Mediation as an example. We present an example illustrating the effects of instructional coaches on beliefs about mathematics being mediated through social network structures.

Significance / Novelty of study

Although social network data are being collected in education and behavioral research, until recently social network modeling has been noticeably absent in large-scale studies. Further, social networks are natural mediators in such studies, but we do not know of any studies in which networks have been used as mediators. We believe there are two reasons for this. First, until recently, there were very few statistical models that could easily accommodate multiple school network data inherent in these studies, and second, there are no known statistical social network models that accommodate a social network as a mediator.

Statistical Model

We propose a framework for modeling networks as mediator in which we fit a statistical social network model and use the parameter from that model to as the mediator in a mediation model. Because we are interested in the effect of an intervention on a network, our unit of observation is at the network level, so social network models that accommodate multiple, independent networks are required.

The single mediator model as introduced by Baron and Kenny (1986) is written as a series of three equations:

$$\begin{aligned} Y &= \beta_1 + \tau X + \epsilon_1 \\ M &= \beta_2 + \alpha X + \epsilon_2 \\ Y &= \beta_3 + \tau' X + \omega M + \epsilon_3 \end{aligned} \tag{0.1}$$

where τ is often called the direct effect (of X on Y). The effect of the X on the mediating variable is α and the effect of M on Y is ω. The parameter τ' represents the effect of X on Y controlling for M.

We propose a framework which essentially replaces M with a network model in Equation (0.1) by building a social network model predicting network ties and using a parameter from that model in Equation (0.1) for M. As an example, we can employ a mixed membership stochastic blockmodel (MMSB; Airoldi et al., 2008) as our mediator.

Suppose A_{ijk} represents the tie from i to j in network k. The full Hierarchical Mixed Membership Stochastic Blockmodel for Mediation is given as

$$\begin{aligned} A_{ijk} &\sim Ber\left({S_{ijk}}^T B R_{jik}\right) \\ S_{ijk} &\sim Multi\left(\theta_{ik}\right) \\ R_{jik} &\sim Multi\left(\theta_{jk}\right) \\ \theta_{ik} &\sim Dir\left(\xi\gamma_k\right) \\ \gamma_k &\sim N(\beta_{01} + \alpha X_k, \sigma_1^2)\ , \gamma_k > 0 \\ Y_k &\sim N(\beta_{02} + \tau' X_k + \omega\gamma_k, \sigma_2^2) \\ \beta_{01} &\sim N(\mu_{01}, {\sigma^2}_{01}) \\ \beta_{02} &\sim N(\mu_{02}, {\sigma^2}_{02}) \\ \alpha &\sim N(\mu_\alpha, {\sigma^2}_\alpha) \\ \tau' &\sim N(\mu_{\tau'}, {\sigma^2}_{\tau'}) \\ \omega &\sim N(\mu_\omega, {\sigma^2}_\omega) \\ \sigma_1^2 &\sim Inv - Gamma(a, b) \\ \sigma_2^2 &\sim Inv - Gamma(c, d)\ , \end{aligned} \tag{0.2}$$

where X, Y, α, τ' and ω are defined as in Equation (0.1); θ_{ik} is person i's probability of belonging to each subgroup or block, S_{ijk} and R_{jik} are the realized subgroup memberships for i and j when i sends a tie to j and B is a block-to-block probability matrix. The parameter of interest is γ_k which determines how insular subgroup structure is in network k. See Airoldi et al. (2008) for additional details of the MMSB and see Sweet and Zheng (2017a,b) for additional details about subgroup structure and the MMSB.

Illustration of the Model

We present an application examining whether teacher advice-seeking networks mediate the effect of introducing coaches into schools and changes in beliefs about instruction. We use data from an elementary school district that has been used regularly in the education research literature (Hopkins et al., 2013).

Throughout the data collection period, 8 schools received mathematics coaches as a way to support and improve mathematics instruction. We expect that beliefs about instruction move in a more positive way in schools with coaches. There are two constructs measuring beliefs about mathematics instruction, procedural beliefs and student-centered beliefs. To construct social networks, we use survey items in which teachers nominated those to whom they sought information and advice regarding mathematics throughout the school year.

Thus, we define Y_k to be the change in procedural or student-centered beliefs about mathematics instruction in one year. X_k is an indicator for whether that school received a coach. The fitted HMMSB for Mediation is given as

$$\begin{aligned}
A_{ijk} &\sim Ber\left({S_{ijk}}^T B R_{jik}\right) \\
S_{ijk} &\sim Multi\left(\theta_{ik}\right) \\
R_{jik} &\sim Multi\left(\theta_{jk}\right) \\
\theta_{ik} &\sim Dir\left(\xi\gamma_k\right) \\
\gamma_k &\sim N(\beta_{01} + \alpha X_k, \sigma_1^2)\ , \gamma_k > 0 \\
Y_k &\sim N(\beta_{02} + \tau' X_k + \omega\gamma_k, \sigma_2^2) \\
\beta_{01} &\sim N(0, 0.1) \\
\beta_{02} &\sim N(0, 3) \\
\alpha &\sim N(0, 3) \\
\tau' &\sim N(0, 3) \\
\omega &\sim N(0, 3) \\
\sigma_1^2 &\sim Inv - Gamma(10, 0.5) \\
\sigma_2^2 &\sim Inv - Gamma(2, 2)\ ,
\end{aligned} \tag{0.3}$$

where a positive Y_k value indicates that beliefs became more procedural (or student-centered) during that year.

Figure 2 shows the summaries of the posterior samples for the parameters of interest in Equation (0.3). Although coaches appear to increase subgroup integration, Figure 2 network subgroup structure appears to mediate the effect of coaches on student-centered beliefs only and not procedural beliefs.

Conclusions

We introduced the Hierarchical Mixed Membership Stochastic Blockmodel for Mediation to model how social network subgroup structure can mediation school interventions. As an illustration, we applied the HMSMB for Mediation to examine the effects of coaches on instructional beliefs and network structure. We found evidence that schools with coaches

have more integrated teacher subgroups (in their advice-seeking networks) and that these integrated subgroups are also associated with teachers becoming more student-centered in their beliefs about instruction. Armed with this information, researchers can better understand how instructional coaches help teachers as well as design future interventions involving coaches.

Appendix A.

References

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Baron, R. M. and Kenny, D. A. (1986). The moderator–mediator variable distinction in social psychological research: Conceptual, strategic, and statistical considerations. *Journal of personality and social psychology*, 51(6):1173.

Hopkins, M., Spillane, J. P., Jakopovic, P., and Heaton, R. M. (2013). Infrastructure redesign and instructional reform in mathematics. *The elementary school journal*, 114(2):200–224.

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Appendix B. Tables and Figures

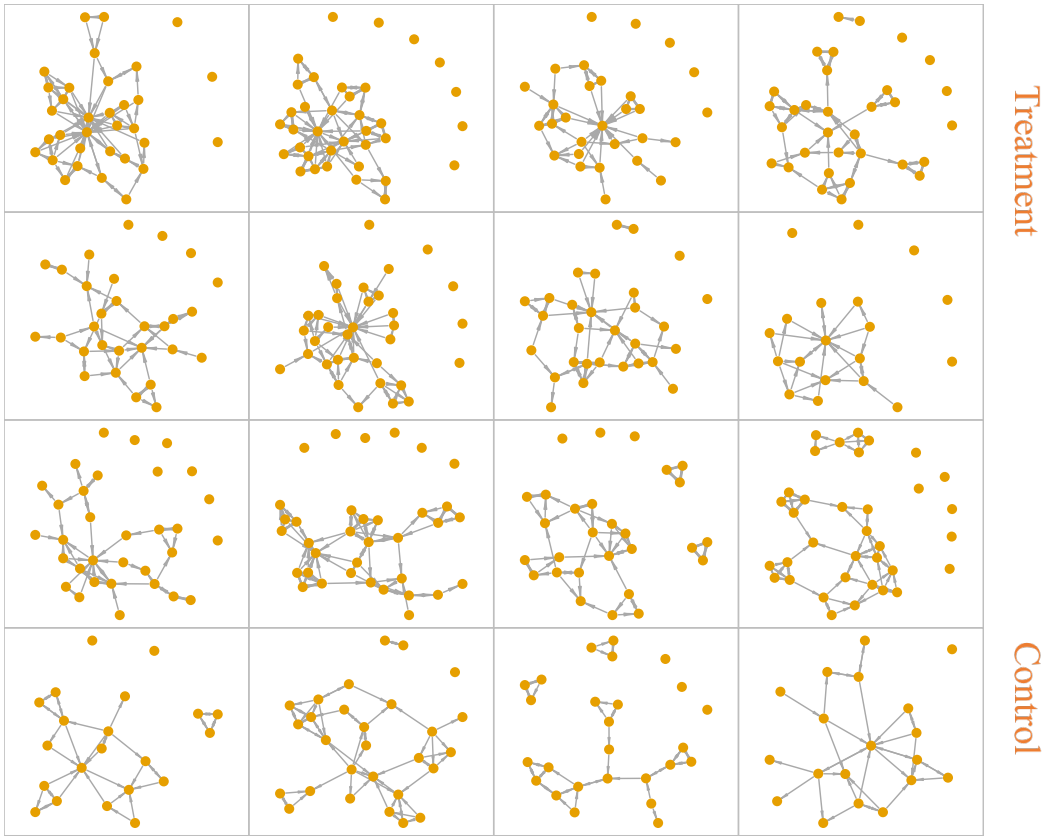

Figure 1: Mathematics advice-seeking networks among teachers in 8 schools the year they received coaches (treatment) and 8 matched schools from the same year who did not receive coaches (control).

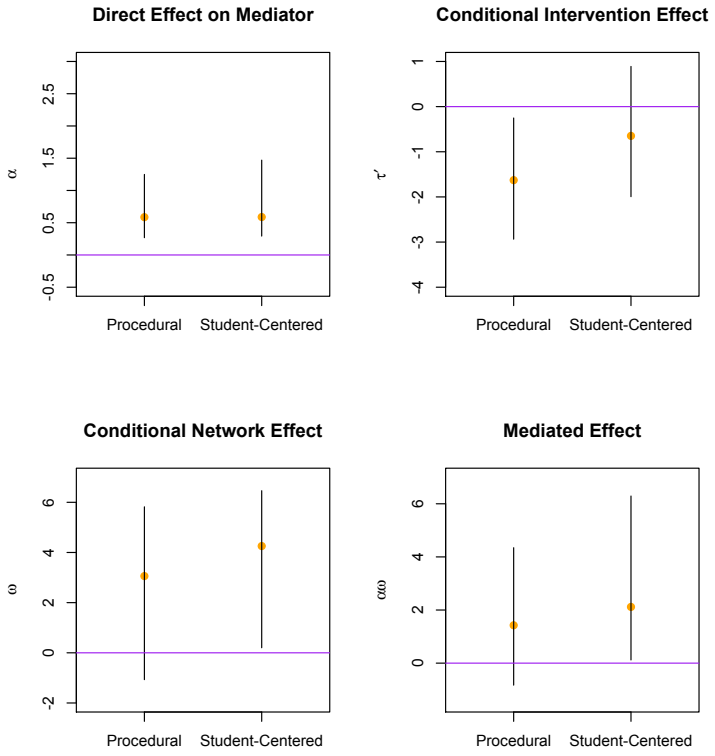

Figure 2: Posterior summaries for HMMSB for Mediation parameters of interest. Posterior modes and 95% equal-tailed credible intervals are given. Results suggest a positive effect of adding coaches on subgroup integration but subgroup structure acts as a mediator on the effect of coaches on changes in student-centered beliefs only.