Trimming of multiple-group propensity score inverse weights: implications for covariate balance and treatment effect estimation

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Introduction

This research focuses on the effects of inverse weight trimming in the context of using multiple-group (MG) propensity scores (PS). McCaffrey and colleagues (2013) introduced the framework for MG PS analysis for estimating treatment effects when there are three or more conditions and when the researcher is interested in the effects of one specific treatment relative to all others. The MG PS approach uses propensity scores inverse weights (i.e., MG IWPS) to balance the covariates among groups and improve the estimation of the treatment effect for the condition of interest. Stuart and colleagues have (2014) showed applications of the MG IWPS for the estimation of treatment effects in the context of difference-in-differences designs.

Prior research on IWPS for two groups has documented the benefits and limitations of IWPS trimming using different PS estimation techniques (Lee, Lessler, & Stuart, 2011). However, no research-to-date has explored the implications of MG IWPS trimming on covariate balance improvement and treatment effect estimation. Thus, it is critical to analyze this topic to understand the role of trimming inverse weights on the estimation of treatment effects. This research focuses on the role of MG IWPS trimming under several simulated propensity score and treatment effect conditions.

Methods

Simulation study

The simulation study began with the random generation of four independent continuous covariates ($X_1,\ldots,X_4$) and two dichotomous covariates (X_5 and X_6) for i=1,…,2000 individual cases. The continuous covariates were generated from independent standard Normal distributions, while the dichotomous covariates from a binomial distribution X_5~B(N=2000, p=0.3) and X_6~B(N=2000, p=0.7). Treated as pretreatment covariates, these six variables had an impact on the formation of four treatment groups $T_i=t \in\{1,\ldots,4\}$ and, subsequently, on a continuous outcome Y_i.

Two factors, each with two levels were considered in this study. The first factor was determined by the multinomial logistic regression model used for the generation of the PS (i.e., a main effects model or a polynomial model with interactions among the covariates), and the second factor by the linear regression model used to generate the outcome (i.e.-, a main effects model or a polynomial model with interactions among the covariates). The combination of these

factors resulted in four different conditions (Table 1); with 250 replications of the experiment conducted in each condition.

For groups t_1 to t_3, the multinomial logistic regression model with only main effects for the MG PS generation was defined as:

$$P(T_i = t) = \exp^{\boldsymbol{\beta}_t \mathbf{X}} \Big/ 1 + \sum_{t=1}^{t=3} \exp^{\boldsymbol{\beta}_t \mathbf{X}}$$

and group t_4

$$P(T_i = 4) = 1 \Big/ 1 + \sum_{t=1}^{t=3} \exp^{\boldsymbol{\beta}_t \mathbf{X}}$$

where $P(T_i)$ is the probability of being assigned to treatment t. The matrix of values for the covariate coefficients $\boldsymbol{\beta}$ were:

	$\beta_1 X_1$	$\beta_2 X_2$	$\beta_3 X_3$	$\beta_4 X_4$	$\beta_5 X_5$	$\beta_6 X_6$
$T = t_1$	0.3	0.2	0	0	0.5	–1.0
$T = t_2$	0	0	0.5	0.5	0	–1.0
$T = t_3$	0.5	0	0	0.5	0	–1.0

The values for $\boldsymbol{\beta}$ were chosen to have a slightly lower proportion of units i in the treatment of interest (i.e., $t = 1$) compared to the other three groups.

A second multinomial logistic regression model for the MG PS included the main effects above and additional squared terms and interactions among covariates.

	$\beta_7 X_1^2$	$\beta_8 X_2^2$	$\beta_9 X_3^2$	$\beta_{10} X_4^2$	$\beta_{11} X_5^2$	$\beta_{12} X_6^2$
$T = t_1$	0.1	0.1	0	0	0.1	–0.1
$T = t_2$	0	0	0.1	0.1	–0.1	–0.1
$T = t_3$	0.1	0	0	0.1	0	–0.1

	$\beta_{13} X_1 X_2$	$\beta_{14} X_1 X_3$	$\beta_{15} X_1 X_4$	$\beta_{16} X_1 X_5$	$\beta_{17} X_1 X_6$	$\beta_{18} X_2 X_3$	$\beta_{19} X_2 X_4$	$\beta_{20} X_2 X_5$
$T = t_1$	0.1	0.1	0	–0.1	–0.1	0.1	0	0
$T = t_2$	0	0	0.1	0	–0.1	0.1	0.1	0
$T = t_3$	0.1	0.1	0.1	–0.1	–0.1	0	0	–0.1

	$\beta_{21}X_2X_6$	$\beta_{22}X_3X_4$	$\beta_{23}X_3X_5$	$\beta_{24}X_3X_6$	$\beta_{25}X_4X_5$	$\beta_{26}X_4X_6$	$\beta_{27}X_5X_6$
$T = t_1$	–0.1	0	0	–0.1	0	–0.1	–0.1
$T = t_2$	–0.1	0.1	0	–0.1	0.1	–0.1	–0.1
$T = t_3$	–0.1	0	–0.1	–0.1	–0.1	–0.1	–0.1

The main effects linear regression model to estimate the treatment effect was:

$$Y_i = \beta_0 + \beta_1 T_{i[1]} + \beta_2 T_{i[2]} + \beta_3 T_{i[3]} + \beta_4 T_{i[4]} + \beta_5 X_1 + \beta_6 X_2 + \beta_7 X_3 + \beta_8 X_4 + \beta_9 X_5 + \beta_{10} X_6 + E_i$$

where the vector of $\boldsymbol{\beta}$ coefficients correspond to the intercept, treatment effects for the four observed conditions $T_{i[t]}$, and covariate effects on the outcome. The error term, E_i, is normally distributed, N(0, 2). The parameter values for the coefficients in this model are $\boldsymbol{\beta} \in \{\beta_0=0, \beta_1=1, \beta_2=-.5, \beta_3=0, \beta_4=-1, \beta_5=.4, \beta_6=.2, \beta_7=-.6, \beta_8=-.3, \beta_9=-.4, \beta_{10}=-.3\}$.

Finally, the linear regression model with polynomials and interactions includes additional coefficients for the interactions among covariates and squared covariate effects on the outcome:

$$\begin{aligned} Y_i = {} & \beta_0 + \beta_1 T_{i[1]} + \beta_2 T_{i[2]} + \beta_3 T_{i[3]} + \beta_4 T_{i[4]} + \beta_5 X_1 + \beta_6 X_2 + \beta_7 X_3 + \beta_8 X_4 + \beta_9 X_5 + \beta_{10} X_6 + \\ & \beta_{11} X_1 X_2 + \beta_{12} X_1 X_3 + \beta_{13} X_1 X_4 + \beta_{14} X_1 X_5 + \beta_{15} X_2 X_3 + \beta_{16} X_3 X_4 + \beta_{17} X_4 X_6 + \\ & \beta_{18} X_3 X_5 + \beta_{19} X_5 X_6 + \beta_{20} X_1^2 + \beta_{21} X_2^2 + \beta_{22} X_3^2 + \beta_{23} X_4^2 + \beta_{24} X_5^2 + \beta_{25} X_6^2 + E_i \end{aligned}$$

with parameter values for the additional coefficients being $\boldsymbol{\beta} \in \{\beta_{11}=.3, \beta_{12}=-.2, \beta_{13}=.1, \beta_{14}=-.3, \beta_{15}=.1, \beta_{16}=.1, \beta_{17}=-.2, \beta_{18}=.3, \beta_{19}=-.3, \beta_{20}=-.2, \beta_{21}=.1, \beta_{22}=.2, \beta_{23}=-.1, \beta_{24}=.1, \beta_{25}=-.2\}$.

In this study, the treatment effect of interest is $T_{[1]}$. Thus, the PS estimation, covariate balance improvement, and weighted treatment effect estimation were done taking $t = 1$ as the group of interest (McCaffrey et al., 2013).

PS and treatment effect estimation

Three different models were used to estimate the MG PS: a main effects multinomial regression model (MLG), a generalized boosted model (GBM), and a neural networks (NN) model (Keller, Kim, & Steiner, 2015; McCaffrey et al., 2013; Stuart et al. 2014). The treatment effect was estimated using an unweighted and *Average Treatment Effects* (ATE) weighted main effects linear regression models leaving out the dichotomous indicator for treatment 4 to prevent multicollinearity. Data generation and statistical analyses were conducted in R (R Core Team, 2016), using 'twang' (Ridgeway, McCaffrey, Morral, Griffin, & Burgette, 2016) and 'nnet' (Venables & Ripley, 2002).

To analyze the role of weight trimming, the three estimated MG IWPS were trimmed based on their percentiles from the 99th to the 50th percentile value, which is the value range explored in prior research on IWPS trimming (Lee et al., 2011). Each one of these trimmed IWPS was used to calculate covariate balance and treatment effect estimates within each

replication. The criterion used to determine covariate balance improvement was the mean maximum absolute covariate bias (McCaffrey et al., 2013). Absolute bias and mean squared error were used as criteria for the treatment effect parameter recovery (Rizzo, 2008).

Results

Table 2 shows that the GBM IWPS percentile distribution is consistent across conditions and it does not produce extreme propensity score weights. However, the MLR and NN approaches estimated some extreme propensity scores in conditions 2 and 4.

The effects of IWPS trimming are represented in Figures 1 and 2. Prior to trimming, GBM IWPS consistently produced the lowest maximum covariate bias across conditions (Figure 1). Conversely, MLR IWPS produces the highest covariate bias in the four conditions explored. IWPS trimming increased covariate bias across conditions. Figure 2 shows that trimming extreme IWPS (e.g., trimming to the 98^{th} percentile) improved the treatment effect estimation, particularly for conditions 2 and 4.

Discussion

This study adds to the current research on the use of MG IWPS estimation and trimming for estimating causal effects in non-randomized designs. More research must be done, particularly regarding omitted covariates impacting the MG PS and treatment effect; prior research on this topic has found that omitted confounders may bias the estimated PS and treatment effects (Austin, 2011; Weitzen, Lapane, Toledano, Hume, & Mor, 2005).

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Tables

Table 1. *Simulation conditions*

Condition	Model for PS generation	Model for outcome generation
1	Main effects multinomial regression model	Main effects linear regression model
2	Multinomial regression model with interactions and squared terms	Main effects linear regression model
3	Main effects multinomial regression model	Linear regression model with interactions and squared terms
4	Multinomial regression model with interactions and squared terms	Linear regression model with interactions and squared terms

Table 2. *Average IWPS percentiles by condition and PS estimation method*

	1st quartile	median	3rd quartile	max
Condition 1				
MLR	1.000	1.000	1.000	1.388
GBM	1.028	1.130	1.427	17.776
NN	1.022	1.091	1.343	13.128
Condition 2				
MLR	1.000	1.000	1.021	4.64×10^{20}
GBM	1.051	1.195	1.548	17.833
NN	1.030	1.124	1.443	143.827
Condition 3				

MLR	1.000	1.000	1.000	1.362
GBM	1.029	1.132	1.429	16.758
NN	1.021	1.089	1.341	14.206
Condition 4				
MLR	1.000	1.000	1.022	8.33×10^{19}
GBM	1.050	1.191	1.542	17.984
NN	1.030	1.125	1.445	180.740

Figures

Figure 1. *Average absolute maximum covariate balance by inverse weight trimming percentile and PS estimation method*

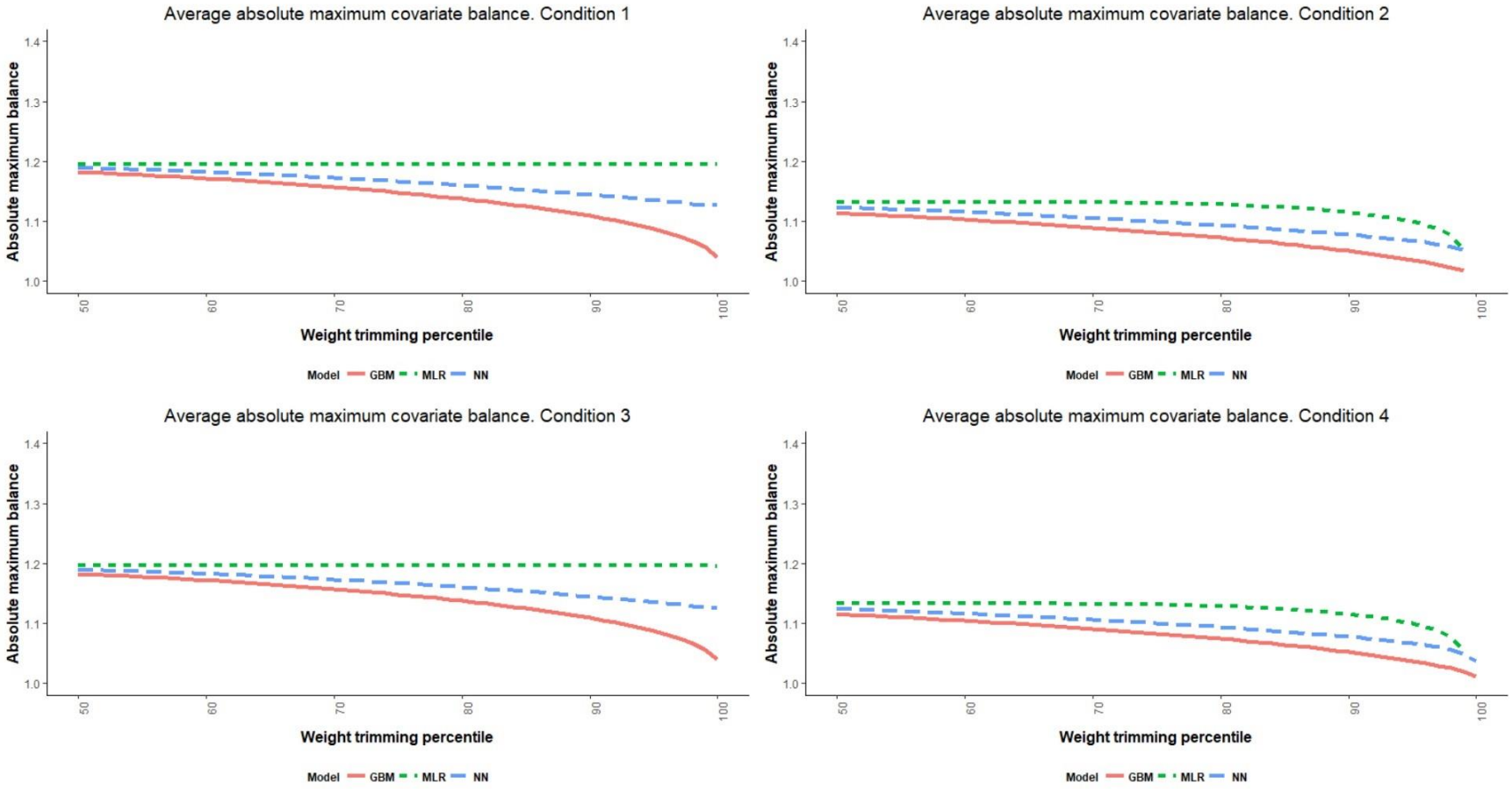

Figure 2. *Treatment effect parameter recovery by inverse weight trimming percentile and PS estimation method*