A Bias-Corrected Limited Information Estimator for Small to Moderate Scale Multilevel Structural Equation Models

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Purpose

Full information maximum likelihood estimation of multilevel structural equation models (MLSEMs) is a preferred approach to operationalize sophisticated theories involving multiple latent variables in clustered or multilevel education settings. Although full information estimators have many desirable properties including consistency, a major limitation is that they often incur significant bias when implemented in studies with a small to moderate number of groups (e.g., 100 or fewer schools). To address similar limitations within the context of single-level studies, recent literature has developed a two-stage limited information estimator that draws on a method of moments type correction that has been shown to yield unbiased estimates in single-level settings, even with small to moderate sample sizes.

In this study, we derive extensions to this estimator for MLSEMs and probe the degree to which the estimator can produce unbiased estimates in small to moderate multilevel samples (e.g., 100 or fewer schools). The results suggest the promise of the estimator—the proposed estimator outperforms full information estimation in terms of bias, error variance, and power when samples include 100 or fewer schools and converges with full information estimation in samples larger than 100 schools. The proposed estimators are implemented in *R* and illustrated through several different types of multilevel mediation examples.

Background

Prior research has shown that the quality of full information estimates of parameters in MLSEMs can be sensitive to the balance between model complexity and sample size because its core properties (e.g., consistency) lean heavily on large sample theory that may not apply to complex models in finite samples (e.g., Li & Beretvas, 2013). For instance, samples of 80 to 100 schools are often considered a minimum in order to produce stable unbiased estimates while samples of less than 80 schools are often classified as precarious (e.g., Kline, 2011; Li & Beretvas, 2013). More carefully, sample size recommendations in past research have often indicated that at least 10 to 20 cases per parameter will be needed to provide a minimal basis for unbiased estimation and inference and even these guidelines can be contingent upon, for example, the strength of the measurement models (e.g., Wolf, Harrington, Clark & Miller, 2013). Yet, many studies in education research draw on sophisticated theories with samples of 100 or few schools (e.g., Spybrook, Shi, & Kelcey, 2016).

Model

To conceptually motivate the need for an alternative estimator, we outline the performance of our proposed estimator using a brief 2-2-1 multilevel mediation example (e.g., Kelcey, Dong, Spybrook, & Shen, 2017). Consider a study with random assignment of schools to an intervention that intends to examine the degree to which the impact of the intervention or treatment (T) on the outcome (Y) operates through a school-level mediator (M) when the outcome, mediator, and covariates are subject to measurement error. We draw on a MLSEM formulation of multilevel mediation that uses a mix of single-level and multilevel common factor models for the covariates, mediator, and outcome and a multilevel structural model that connects these latent variables (see Figure 1 for summary).

Estimator

In our omitted derivations, we derive a Croon (2002) type two-stage corrected estimator that estimates the covariances among the latent variables in step one and then adjusts those covariances for bias stemming from the measurement uncertainty. To derive the new estimator, we used a method of moments approach that equates the sample and population covariances. As a simple example of the resulting corrections, consider the covariance between the latent school-level mediator ($\tilde{M}$) and the school-level component of the latent outcome ($\tilde{Y}^{L2}$). Based on our derivations, the resulting correction is

$$\operatorname{cov}(\eta_Y^{L2}, \eta_M) = \frac{\operatorname{cov}(\tilde{Y}_{EB}^{L2}, \tilde{M})}{\mathbf{A}_Y^{L2} \boldsymbol{R}_y \Lambda_Y^{L2} \Lambda_M^{'} \mathbf{A}_M^{'}} \tag{1}$$

where $\operatorname{cov}(\eta_Y^{L2}, \eta_M)$ is the population covariance between the school-level component of the outcome and the mediator, $\operatorname{cov}(\tilde{Y}^{L2}, \tilde{M})$ is the sample covariance, $\mathbf{A}$ and Λ are the factor score and loading matrices, and $\boldsymbol{R}_y$ is a vector of outcome indicator reliabilities.

Simulation

To provide an initial assessment of the value and precision of the estimator developed, we generated multiple sets of Monte Carlo simulations using multilevel mediation models. We report a sample of these simulation results to outline the performance relative to the full information estimator for 2-2-1 (Figure 1) and 2-1-1 (Figure 2) multilevel mediation examples and highlight two criteria: (a) parameter bias, (b) parameter dispersion (relative magnitude of standard errors), and (c) bias of standard errors (Tables 1-3). The results demonstrated that the proposed estimator widely outperformed the full information estimator. For example, in Tables 1 and 2 we see that the average bias across the sampled conditions for the new estimator was nearly zero for both paths but the average bias for the full information estimator was massive for the mediator-outcome path (over 1000% relative bias) and 0.11 for the intervention-mediator (33% relative bias) paths. Collectively, the results suggest that the new limited information estimator performs quite well small to moderate samples while repeating the findings of prior studies that suggest that the full information estimator is unstable and biased in small to moderate samples.

Conclusion

Education literature has recognized that the benefits of empirical studies are not limited to only large scale studies—small to moderate scale studies can also offer critical contributions to theory and practice when they are well executed (e.g., Walton, 2014; Bodner & Bliese, 2017). Yet, many studies in education that draw on small to moderate samples (e.g., 100 or fewer schools) also draw on sophisticated theories that require MLSEMs. This combination—small/moderate school sample size coupled with sophisticated MLSEMs—poses significant challenges for full information estimation methods because stable and unbiased estimates under this method demand a large sample-to-parameter ratio. For this reason, we developed an alternative bias-corrected limited information estimator. The results of our study suggest that with samples of less than about 100 schools, the proposed limited information estimator is likely to outperform the conventional full information estimator in terms of bias, error variance, and power.

Appendices

Appendix A. References

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Appendix B. Tables and Figures

Figure 1
Conceptual representation of an example 2-2-1 MLSEM mediation model when the mediator, outcome, and covariate are subject to measurement error.

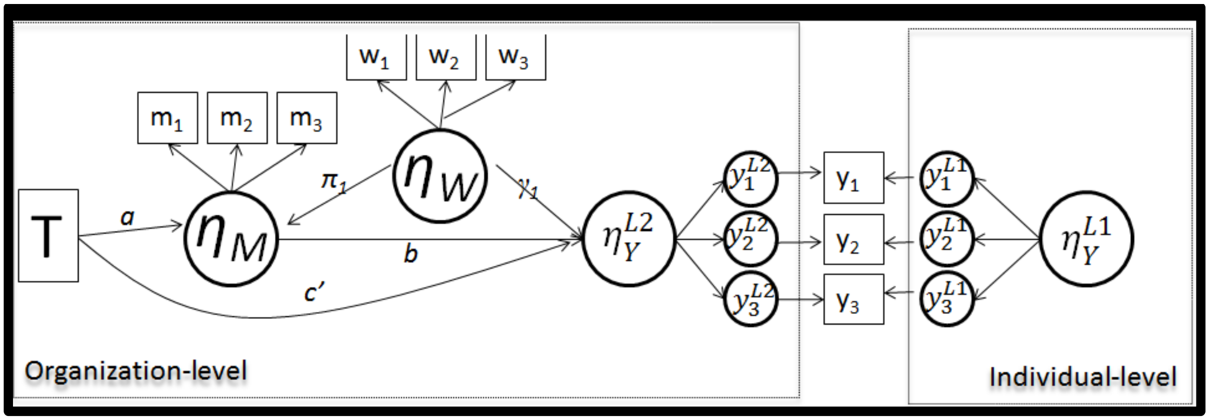

Figure 2
Conceptual representation of an example 2-1-1 MLSEM mediation model when the mediator, outcome, and covariates are subject to measurement error.

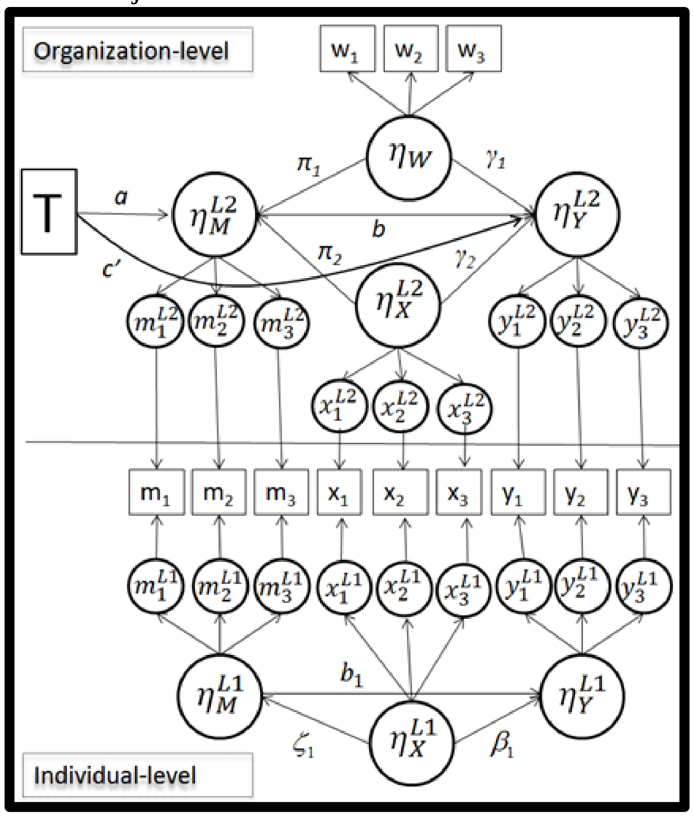

Table 1
Bias by Condition and Method for 2-2-1 Mediation

Condition	Sample Size		Variance Explained		Mediator-Outcome Path Coefficient		Intervention-Mediator Path Coefficient	
	n_2	n_1	R_y^2	R_m^2	FI	BC	FI	BC
1	100	20	0.6	0.6	0.15	-0.01	0.09	0.00
2	30	20	0.6	0.6	2.82	-0.04	0.16	-0.01
3	40	20	0.6	0.6	0.90	-0.02	0.13	-0.01
4	50	20	0.6	0.6	0.22	-0.01	0.12	0.00
5	30	5	0.6	0.6	131	-0.08	0.14	-0.01
6	30	50	0.8	0.6	0.09	-0.00	0.19	-0.02
7	50	50	0.8	0.6	0.04	-0.01	0.05	-0.01
8	50	50	0.8	0.8	0.03	-0.01	0.05	0.00
9	30	10	0.9	0.9	22	-0.03	0.15	0.01
10	30	100	0.5	0.8	15	-0.02	0.17	0.00
11	40	30	0.8	0.5	0.08	-0.02	0.07	-0.03
12	40	30	0.5	0.5	0.07	-0.03	0.07	-0.03
13	40	5	0.5	0.5	0.07	-0.04	0.08	-0.02
14	40	5	0.5	0.8	0.11	-0.03	0.07	0.00
15	40	20	0.8	0.8	0.14	0.00	0.15	0.00
16	40	20	0.8	0.8	0.21	-0.01	0.24	0.00
17	60	20	0.8	0.8	0.02	0.00	0.02	0.00
		Average Absolute Bias			10	-0.02	0.11	-0.01

Note. FI is full information maximum likelihood; BC is bias-corrected limited information. R_y^2 and R_m^2 are proportion of indicator variation explained at each level by the latent factor.

Table 2 Bias by Condition and Method for 2-1-1 Mediation

Condition	Sample Size		ICC		Variance Explained		Mediator-Outcome Path Coefficient		Intervention-Mediator Path Coefficient	
	n_2	n_1	ρ_y	ρ_m	R_y^2	R_m^2	FI	BC	FI	BC
1	100	20	0.30	0.15	0.60	0.60	1.89	0.06	4.74	-0.02
2	30	20	0.30	0.15	0.60	0.60	60.99	-0.07	63.93	-0.06
3	40	20	0.30	0.15	0.60	0.60	23.20	-0.02	43.64	-0.04
4	50	20	0.30	0.15	0.60	0.60	12.35	0.02	32.60	-0.02
5	30	5	0.30	0.15	0.60	0.60	135.3	-0.16	246.4	-0.27
6	30	50	0.30	0.15	0.80	0.60	13.28	0.02	62.17	-0.03
7	50	50	0.30	0.15	0.80	0.60	2.55	0.02	18.81	-0.02
8	50	50	0.30	0.15	0.80	0.80	0.27	0.05	9.17	0.00
9	30	10	0.20	0.10	0.90	0.90	233.7	0.02	137.3	-0.05
10	30	100	0.20	0.30	0.50	0.80	13.05	-0.02	0.11	0.00
11	40	30	0.20	0.20	0.80	0.50	1.72	-0.01	0.77	-0.04
12	40	30	0.70	0.70	0.50	0.50	1.03	-0.04	0.10	-0.03
13	40	5	0.70	0.70	0.50	0.50	3.24	-0.03	0.11	-0.03
14	40	5	0.50	0.50	0.50	0.80	5.89	-0.01	0.10	0.00
15	40	20	0.50	0.50	0.80	0.80	0.11	0.02	0.16	0.00
16	40	20	0.30	0.50	0.80	0.80	1.64	0.02	0.15	0.00
17	60	20	0.10	0.15	0.80	0.80	0.05	0.04	0.03	0.00
				Average Absolute Bias			30.01	0.04	36.48	0.04

Note. FI is full information maximum likelihood; BC is bias-corrected limited information. R_y^2 and R_m^2 are proportion of indicator variation explained at each level by the latent factor. ρ_m and ρ_y are the intraclass correlation coefficients for the mediator and outcome indicators.

Table 3: *Standard Errors by Condition and Method*

Condition	n_2	n_1	FI SD	FI SE	BC SD	BC SE	Boot
Mediator-Outcome Path							
1	100	20	0.24	0.21	0.12	0.08	0.12
2	30	20	13.15	15.86	0.23	0.15	0.25
3	40	20	19.06	3.53	0.21	0.12	0.21
4	50	20	0.88	0.90	0.18	0.11	0.18
5	30	5	2686	178	0.31	0.15	0.30
6	30	50	1.17	1.50	0.21	0.14	0.22
7	50	50	0.23	0.21	0.18	0.14	0.19
8	50	50	0.20	0.19	0.16	0.14	0.17
9	30	10	7.65	25	0.25	0.13	0.25
10	30	100	141.79	20	0.22	0.15	0.23
11	40	30	0.37	0.31	0.23	0.15	0.24
12	40	30	0.33	0.28	0.21	0.15	0.23
13	40	5	4.59	0.43	0.21	0.15	0.23
14	40	5	0.74	0.76	0.21	0.15	0.21
15	40	20	0.29	0.25	0.16	0.12	0.16
16	40	20	0.48	0.42	0.16	0.13	0.17
17	60	20	0.21	0.19	0.17	0.12	0.17
Average			169.29	14.64	0.20	0.13	0.21
Intervention-Mediator Path							
1	100	20	0.25	0.24	0.20	0.18	0.20
2	30	20	0.56	0.48	0.38	0.31	0.38
3	40	20	0.45	0.40	0.32	0.27	0.32
4	50	20	0.37	0.35	0.28	0.25	0.29
5	30	5	0.57	0.47	0.37	0.32	0.37
6	30	50	0.52	0.46	0.36	0.32	0.38
7	50	50	0.38	0.33	0.32	0.26	0.31
8	50	50	0.33	0.31	0.28	0.27	0.29
9	30	10	0.43	0.40	0.31	0.29	0.32
10	30	100	0.47	0.42	0.32	0.30	0.33
11	40	30	0.45	0.40	0.35	0.29	0.35
12	40	30	0.46	0.40	0.35	0.29	0.35
13	40	5	0.46	0.41	0.35	0.29	0.35
14	40	5	0.38	0.35	0.32	0.29	0.32
15	40	20	0.39	0.36	0.28	0.26	0.29
16	40	20	0.40	0.37	0.26	0.26	0.28
17	60	20	0.28	0.28	0.26	0.25	0.27
Average			0.42	0.38	0.31	0.28	0.32

Note. FI is full information maximum likelihood; BC is bias-corrected limited information. SD is the empirical standard deviation of the point estimate across simulation draws, SE is the model-based standard error based on the observed information and boot is the bootstrap-based estimate.