Comparing a Learning Trajectory Approach to a Teach-to-the-Target Approach in Early Arithmetic

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1. Objectives

Learning trajectories (LTs) have three parts: (1) a goal, (2) a developmental progression, and (3) instructional activities. To attain a certain mathematical competence in a given topic or domain (the goal), children learn each successive level (the developmental progression), aided by tasks (instructional activities) designed to build the mental actions-on-objects that enable thinking at each higher level (Author & Author, 2004). LTs have received increasing attention (Maloney, Confrey, & Nguyen, 2014; Author & Author, 2009) and are deemed as a useful tool for standards development and organization, instruction, and student assessment (Frye et al., 2013; National Research Council, 2009). However, there is little or no empirical evidence of the *efficacy* of such an approach to teaching. Thus, we have designed multiple experiments comparing an LT-based intervention to an approach that is similar in every way to the LT approach except that one critical attribute of an LT is altered. In this experiment, we ran a pilot to compare the learning of Kindergarteners who received instruction on arithmetic—including support in prerequisite counting—based on an empirically-validated LT for arithmetic to those who received an equal amount of instruction focused only on target levels (thus, essentially skipping instruction at intermediate levels).

2. Theoretical framework

Although the results of this pilot study will have implications for both a larger scale arithmetic LT efficacy study and the use of LTs across multiple domains (e.g., Alonzo & Gotwals, 2012; National Research Council, 2007), the domain of early mathematics is particularly important and fecund for this research. LTs have played a substantial role in mathematics education (CCSSO/NGA, 2010; NRC, 2009; NCTM, 2006; Simon, 1995) and have been successfully applied in early mathematics intervention projects. These studies support the efficacy of the use of LTs but have not identified their unique contribution (Clarke et al., 2001; Fantuzzo, Gadsden, & McDermott, 2011; Gravemeijer, 1999; Jordan etal., 2012).

3. Methods

We used an experimental research design, to answer the following overarching research question: Does instruction in which arithmetic LT levels are taught consecutively (for children at level n, instructional tasks from level n + 1, then n + 2) result in greater learning than instruction that immediately and solely targets level n + 3 (or more, the “skip levels” approach)? Are (a) entering knowledge, (b) ethnic group, or (c) children’s age significant moderators of differences?

4. Data sources, evidence

Participants

We recruited and received consent from all 25children enrolled in two kindergartens; 11 were female and 14 male; one was Asian, 2 Black or African American, 2 Hispanic and 20 White/Not Hispanic or Latino; average 5.45 years old.

Procedures

The LT group received nine 20-minute sessions based on an empirically-validated LT for arithmetic such that each session was offered at the level directly following the level the child has mastered (see Table 1). The skip group received nine sessions focused on particular target LT levels, skipping over 2 LT levels from their initial level of mastery to the target level (see Table 2), based on research and practice that promote providing accurate definitions and demonstrating accurate mathematical procedures using direct instruction (Carnine, Jitendra, & Silbert, 1997; Clark, Kirschner, & Sweller, 2012; Wu, 2011).

Measures

The pre-assessment and post-assessment was comprised of counting and arithmetic items from the REMA (Authors, 2008/2016) and TEMA-3 (Authors, 2003). The counting assessment items reflected prerequisite knowledge for early levels of arithmetic, such as, verbal and object counting, correspondence, cardinality, and small set production. The counting assessment items were given to all student participants separate from and the day before the arithmetic assessment items.

Analyses

We analyzed pre and post-assessments to compare growth of LT levels in arithmetic (and counting) for each child (see Tables 1 and 2 and Figures 1 and 2). We also longitudinally traced all children in the LT treatment, to compare instructor "LT leveling" at the session level to assessment performance (Figure 3).

5. Results

Eight LT children grew at least one arithmetic LT level from pre to post-assessment (see Table 1 and Figure 1). Four children were leveled as "Small Number +/-" on the arithmetic LT but showed no arithmetic knowledge and needed support on the counting LT to make progress in arithmetic (noted in Table 1). One of those children (03) grew only in counting knowledge, and not on the arithmetic LT. Four other children did not show pre- to post- growth. Analyses of the instructors session-by-sessions revealed that instructors observed children to be at higher LT levels during instruction, than as evidenced on the post-assessment (for example see Figure 3). Only one skip child grew from pre to post-assessment (see Table 2 and Figure 2).

6. Discussion and Scholarly Significance

LT instruction was somewhat more effective than skip-levels instruction based on pre-post but was substantially more effective based on instructor analyses of children's LT levels during instructional sessions. We are presently analyzing the pre and post-assessment data to incorporate not just item correctness, but also strategy use. Skip-levels instruction did not

generally lead to children's growth, even when instruction "backed up" from $n + x$ ($x > 2$) to as close as $n + 2$.

LT-based approaches justify the assumption that each contiguous level be taught consecutively because LT's developmental progressions are more than linear sequences based on accretion of numerous facts and skills. Each is based on a progression of levels of thinking that reflects the cognitive science view of knowledge as interconnected webs of concepts and skills. Each level is characterized by specific mental actions-on-objects hypothetically must be built at level n as a foundation for successful learning of all subsequent levels. Thus, the critical question, "Is it necessary to teach each contiguous level of an LT or can instruction be equally effective when skipping levels, provided the necessary exemplars are made?" must be answered. We believe that each level of instruction is critically necessary for the successful acquisition of subsequent information, as it provides an anchor and firm support from which to build new knowledge.

Table 1

Class 1 Arithmetic: pre-assessment LT levels, counting instruction notes, anticipated post-assessment LT levels, and post-assessment LT levels

Child	Pre-Assessment LT Level	Counting Support Needed	Anticipated Post-Assessment LT Level	Post-Assessment LT Level
1	Small Number +/-	Yes	Part Whole +/-	Find Result +/-
2	Find Result +/-	No	Find Change +/-	Find Result +/-
3	Small Number +/-	Yes	Find Result +/-	Small Number +/-
4	Small Number +/-	No	Find Change +/-	Find Result +/-
5	Small Number +/-	No	Numbers-in-Numbers +/-	Small Number +/-
6	Small Number +/-	Yes	Find Change +/-	Find Result +/-
7	Small Number +/-	No	Counting Strategies +/-	Find Result +/-
8	Find Result +/-	No	Part Whole +/-	Find Result +/-
9	Small Number +/-	No	Problem Solver +/-	Find Result +/-
10	Small Number +/-	Yes	Part Whole +/-	Find Change +/-
11	Small Number +/-	No	Find Change +/-	Find Result +/-
12	Find Result +/-	No	Deriver +/-	Find Result +/-
13	Small Number +/-	No	Numbers-in-Numbers +/-	Find Result +/-

Table 2

Class 2 Arithmetic: pre-assessment LT levels, target instructional LT levels, post-assessment LT levels

Child	Pre-Assessment LT Level	Target LT Level 1 Problem Type Problem Size	Target LT Level 2 Problem Type Problem Size	Target LT Level 3 Problem Type Problem Size	Post-Assessment LT Level
14	Small Number +/-*	Numbers-in-Numbers +/- Start/Part Unknown 20-100	Numbers-in-Numbers +/- Start/Part Unknown 0-20	Find Change +/- Change/Difference Unknown 0-10	Small Number +/-*
15	Small Number +/-*	Numbers-in-Numbers +/- Start/Part Unknown 20-100	Find Change +/- Change/Difference Unknown 20-100	Find Result +/- Result/Whole Unknown 20-100	Small Number +/-*
16	Small Number +/-	Numbers-in-Numbers +/- Start/Part Unknown 20-100	Find Change +/- Change/Difference Unknown 20-100	Find Result +/- Result/Whole Unknown 20-100	Find Result +/-
17	Small Number +/-*	Numbers-in-Numbers +/- Start/Part Unknown 20-100	Numbers-in-Numbers +/- Start/Part Unknown 0-10	Find Change +/- Change/Difference Unknown 20-100	Small Number +/-*
19	Small Number +/-	Numbers-in-Numbers +/- Start/Part Unknown 20-100	Numbers-in-Numbers +/- Start/Part Unknown 0-10	Find Change +/- Change/Difference Unknown 20-100	Small Number +/-
20	Small Number +/-*	Numbers-in-Numbers +/- Start/Part Unknown 20-100	Find Change +/- Change/Difference Unknown 20-100	Find Change +/- Change/Difference Unknown 0-10	Small Number +/-*
21	Small Number +/-	Numbers-in-Numbers +/- Start/Part Unknown 20-100	Numbers-in-Numbers +/- Start/Part Unknown 20-100	Find Result +/- Result/Whole Unknown 20-100	Small Number +/-

22	Small Number +/-	Numbers-in-Numbers +/- Start/Part Unknown 20-100	Find Change +/- Change/Difference Unknown 0-10	Find Change +/- Change/Difference Unknown 0-10	Small Number +/-*
23	Small Number +/-	Numbers-in-Numbers +/- Start/Part Unknown 20-100	Numbers-in-Numbers +/- Start/Part Unknown 20-100	Find Change +/- Change/Difference Unknown 20-100	Small Number +/-
24	Small Number +/-	Numbers-in-Numbers +/- Start/Part Unknown 20-100	Numbers-in-Numbers +/- Start/Part Unknown 0-10	Find Change +/- Change/Difference Unknown 0-10	Small Number +/-
25	Small Number +/-	Numbers-in-Numbers +/- Start/Part Unknown 20-100	Find Change +/- Change/Difference Unknown 20-100	Find Change +/- Change/Difference Unknown 0-10	Small Number +/-
26	Small Number +/-	Numbers-in-Numbers +/- Start/Part Unknown 20-100	Numbers-in-Numbers +/-Start/Part Unknown 0-10	Find Change +/- Change/Difference Unknown 20-100	Small Number +/-

*Children who would have received counting support in an LT treatment.

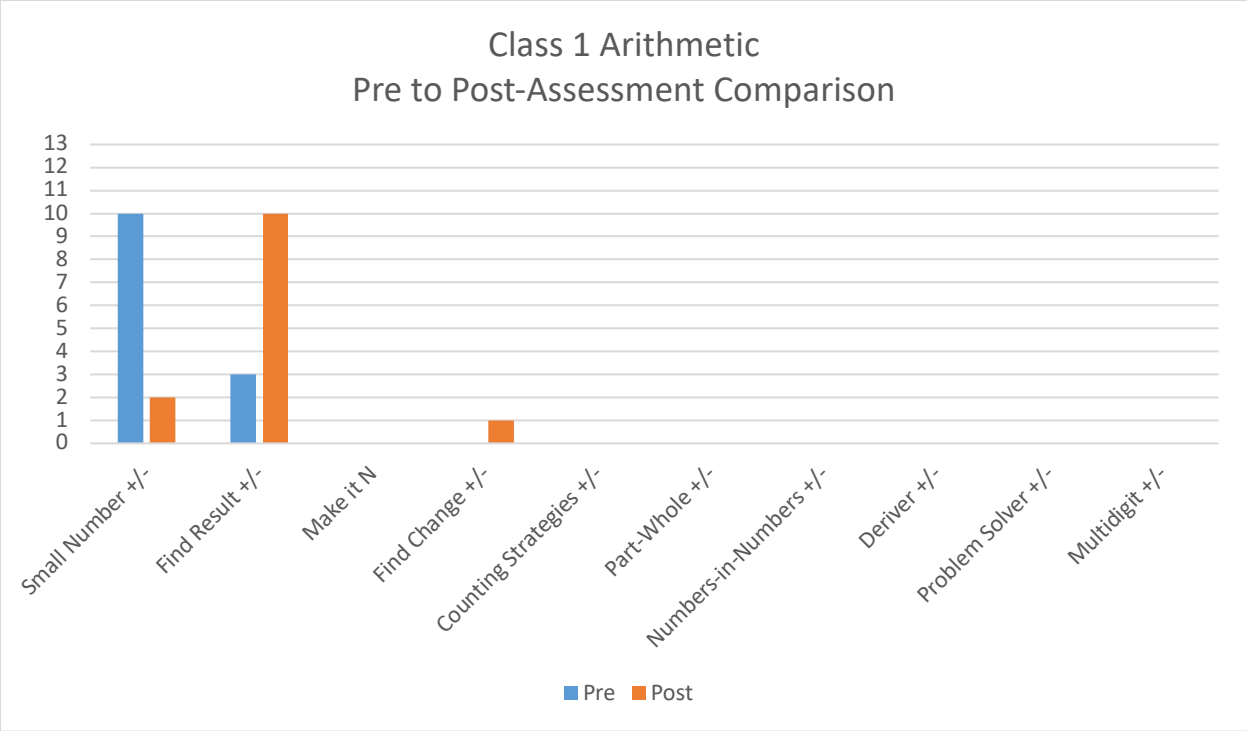

Figure 1. Class 1 Arithmetic: Pre to post-assessment comparison.

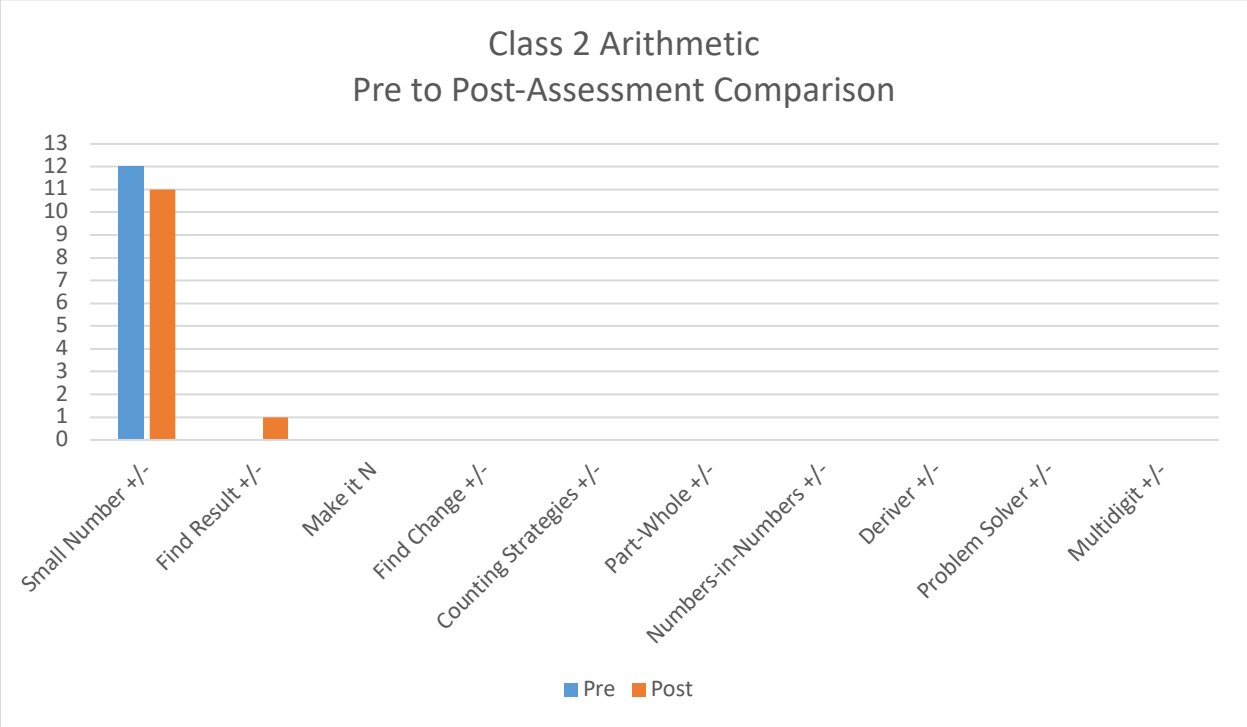

Figure 1. Class 2 Arithmetic: Pre to post-assessment comparison.

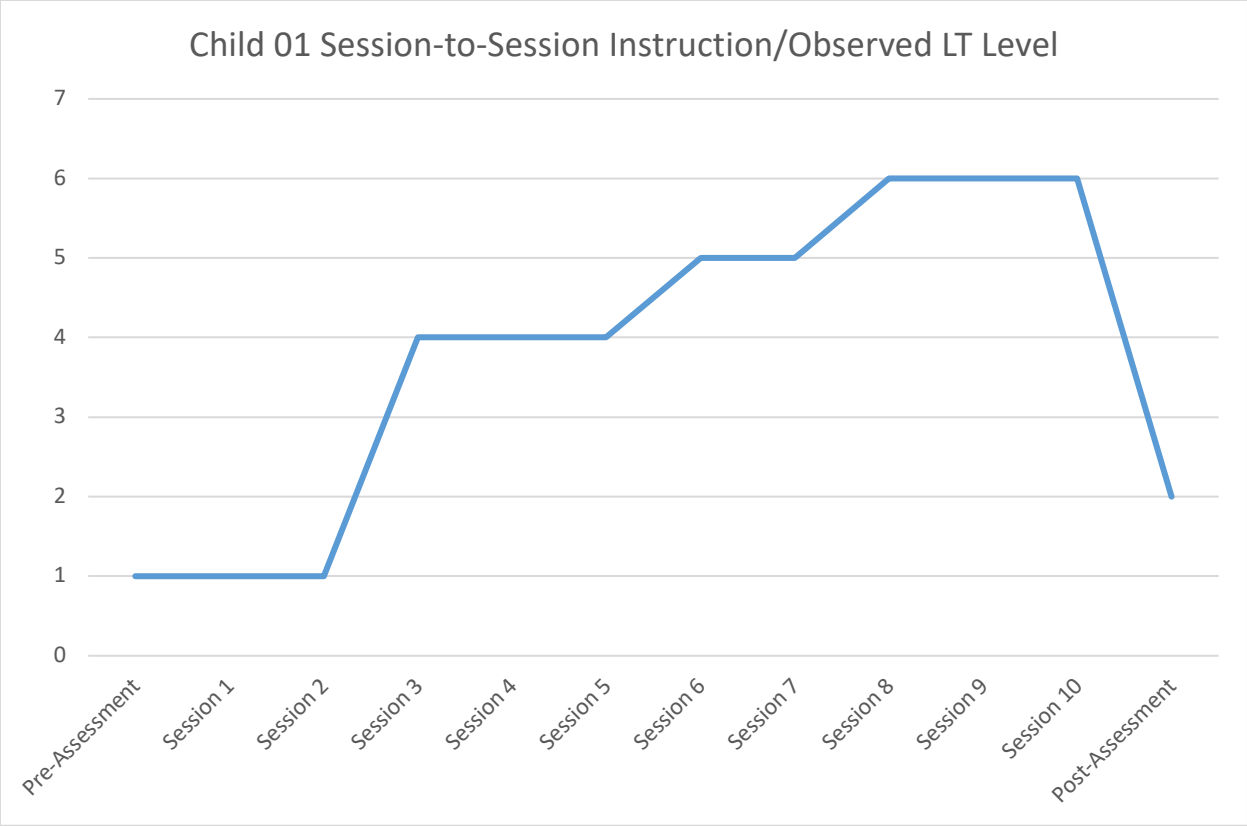

Figure 3. Child 01 Arithmetic: Session-to-session instruction/observed LT levels.

References

Alonzo, A. C., & Gotwals, A. W. (2012). Leaping forward: Next steps for learning progressions in science. In A. C. Alonzo & A. W. Gotwals (Eds.), *Learning Progressions in Science* (pp. 475-490): Sense Publishers.

Casey, B. M., Erkut, S., Ceder, I., & Young, J. M. (2008). Use of a storytelling context to improve girls' and boys' geometry skills in kindergarten. *Journal of Applied Developmental Psychology, 29*, 29-48.

CCSSO/NGA. (2010). *Common core state standards*. Washington, DC: Council of Chief State School Officers and the National Governors Association Center for Best Practices.

Clarke, D. M., Cheeseman, J., Clarke, B., Gervasoni, A., Gronn, D., Horne, M., . . . Sullivan, P. (2001). Understanding, assessing and developing young children's mathematical thinking: Research as a powerful tool for professional growth. In J. Bobis, B. Perry & M. Mitchelmore (Eds.), *Numeracy and beyond (Proceedings of the 24th Annual Conference of the Mathematics Education Research Group of Australasia, Vol. 1)* (pp. 9-26). Reston, Australia: MERGA.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9-13.

Author, & Author (2011).

Fantuzzo, J. W., Gadsden, V. L., & McDermott, P. A. (2011). An integrated curriculum to improve mathematics, language, and literacy for Head Start Children. *American Educational Research Journal, 48*, 763-793.

Gravemeijer, K. P. E. (1999). How emergent models may foster the constitution of formal mathematics. *Mathematical Thinking and Learning, 1*, 155-177.

Jordan, N. C., Glutting, J., Dyson, N., Hassinger-Das, B., & Irwin, C. (2012). Building kindergartners' number sense: A randomized controlled study. *Journal of Educational Psychology, 104*(3), 647-660. doi: 10.1037/a0029018

Maloney, A. P., Confrey, J., & Nguyen, K. H. (Eds.). (2014). *Learning over time: Learning trajectories in mathematics education*. New York, NY: Information Age Publishing.

Mansfield, H. M., & Scott, J. (1990). Young children solving spatial problems. In G. Booker, P. Cobb & T. N. deMendicuti (Eds.), *Proceedings of the 14th annual conference of the Internation Group for the Psychology of Mathematics Education* (Vol. 2, pp. 275-282). Oaxlepec, Mexico: Internation Group for the Psychology of Mathematics Education.

Murata, A. (2004). Paths to learning ten-structured understanding of teen sums: Addition solution methods of Japanese Grade 1 students. *Cognition and Instruction, 22*, 185-218.

National Research Council. (2007). *Taking science to school: Learning and teaching sciences in grades K-8*. Washington, DC: National Academies Press.

National Research Council. (2009). *Mathematics learning in early childhood: Paths toward excellence and equity*. Washington, DC: National Academy Press.

NCTM. (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics: A quest for coherence*. Reston, VA: National Council of Teachers of Mathematics.

Platt, J. R. (1964). Strong inference. *Science, 146*(3642), 347-353.

Sales, C. (1994). *A constructivist instructional project on developing geometric problem solving abilities using pattern blocks and tangrams with young children.* (Masters), University of Northern Iowa, Cedar Falls, Iowa.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*(2), 114-145. doi: 10.2307/749205

Wright, R. J., Stanger, G., Stafford, A. K., & Martland, J. (2006). *Teaching number in the classroom with 4-8 year olds*. London: Paul Chapman/Russell Sage.