Controlling for latent confounding by confirmatory factor analysis (CFA)

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Background

Latent confounder is common in social and behavioral science in which most of cases the selection mechanism is neither fully known nor perfect measured. To measure latent confounder, multiple indicators need to be included to enhance both reliability and validity of the measurement. But, condition on many covariates may cause estimation problem or inefficiency. In this paper, we will investigate how confirmatory factor analysis (CFA) could be used for controlling for selection bias caused by a latent confounding variable.

Methods for Latent Confounding Control

Confirmatory factor analysis (CFA)

Confirmatory factor analysis (CFA) incorporates the subjective knowledge of latent structure, observed measurements and measurement errors to estimate latent variables (Bollen, 1989). CFA derives latent variable as linear combinations of independent observed variables. The factor loadings indicate the relative importance of each factor to the latent variable. Factor scores are composite score that provide information about each unit's predicted placement on the latent factors. In this paper, we use Bartlett Scores which have the advantage of producing unbiased estimates of true factor scores (Hershberger, 2005).

Data-Based Covariate Selection

We define covariate selection as the process by which a subset $X_s \subseteq$ X identified with the aim of satisfying ignorability. We use the theoretical framework proposed by de Luna et al. (2011) referred to as X_Y because it retains outcome-only predictors, which increases efficiency (Brookhart et. al, 2006). To implement conditional independence testing, we used max-min parents and children algorithm for Gaussian Bayesian networks (GBN) to select target covariates (Scutari, 2010).

Hybrid Approach and Kitchen Sink Approach

The hybrid approach is a combination of covariate selection and factor analysis insofar as it involves conditioning on the covariate set selected by covariate selection together with the estimated factor score. The rationale for developing this approach is that it avoids information loss of factor analysis and simultaneously prevents bias caused by omitting important covariates missed by covariate selection, while still allowing for dimension reduction of the covariate space. The kitchen sink approach involves including all measured pre-treatment covariates without any pre-processing; this approach will be set as benchmark for comparison in our study.

Research Questions

The three motivation questions for this simulation studies are as follow.

1. Is it possible for confirmatory factor analysis (CFA) to successfully control for selection bias caused by latent confounders? Furthermore, how do sample size and factor structure and loadings contribute?
2. How does CFA compare to other methods with respect to bias and efficiency?
3. What are sufficient conditions to successfully implement CFA for latent confounding control?

Monte Carlo Simulation Design

The data generating process was motivated by the model specification for CFA. Data were generated as follows:

$$F^1 = N(0,1)$$
$$X^1 = a_1 \times F^1 + \epsilon_1$$
$$X^2 = a_2 \times F^1 + \epsilon_2$$
$$\ldots.$$
$$X^{10} = a_{10} \times F^1 + \epsilon_{10}$$

$$\log\left(\frac{PS}{1-PS}\right) = b_0 + b_1 \times F^1$$
$$Z = Bernoulli(PS)$$
$$Y^1 = \beta_o + 2 + \beta_1 \times F^1 + \epsilon^1$$
$$Y^0 = \beta_o + \beta_1 \times F^1 + \epsilon^0$$

Where $\epsilon_{i=1\ldots.10} \sim N(0, \sqrt{1-a_i^2})$ and $\epsilon^1, \epsilon^0 \sim N(0,1)$

See Figure 1. for a graphical representation. The confounding coefficients are set as follows: $b_0 = 0, b_1 = 2$ and $\beta_0 = 0, \beta_1 = 2$.

The simulation design factors are:

- Sample sizes: 100, 500, 1000.
- Strengths of factor loadings.

(1). Strong vs. weak

$$[a_1 = 0.9, a_2 = 0.1 \ldots., a_{10} = 0.1],$$
$$[a_1 = 0.9, a_2 = 0.9 \ldots., a_{10} = 0.1],$$
$$\ldots.$$
$$[a_1 = 0.9, a_2 = 0.9, \ldots., a_{10} = 0.9].$$

(2). Medium vs. weak

$$[a_1 = 0.5, a_2 = 0.1 \ldots., a_{10} = 0.1],$$
$$[a_1 = 0.5, a_2 = 0.5 \ldots., a_{10} = 0.1],$$
$$\ldots.$$

$$[a_1 = 0.5, a_2 = 0.5, \ldots., a_{10} = 0.5].$$

(3). Small vs. weak

$$[a_1 = 0.3, a_2 = 0.1 \ldots., a_{10} = 0.1],$$
$$[a_1 = 0.3, a_2 = 0.3 \ldots., a_{10} = 0.1],$$
$$\ldots.$$
$$[a_1 = 0.3, a_2 = 0.3, \ldots., a_{10} = 0.3].$$

The analysis factor is based on the four different approaches to deal with a latent confounder: covariate selection, confirmatory factor analysis, hybrid method and kitchen-sink approach. The primary simulation outcomes are the bias and MSE of regression treatment effect estimation. We also quantify the number of covariates selected with the covariate selection method. 100 replications were run for each cell of the study.

Simulation Results

Results of the simulation study are reported in Tables 1 to 9. The true value of the treatment effect was 2 units (cf., Figure 1). The overall sample size, which was varied from 100 to 500 to 1000, did not have an appreciable influence on bias reduction.

There is a clear interaction between the strength of the "strong" factor loadings and the number of strong factor loadings. When the "strong" loadings are all set to 0.3 (Table 4), the bias ranges from about 2.3 (or 115% of the treatment effect), when none of the indicators is strong, to about 1.5 (75%), when all of them are. It is worth noting that a standardized loading of 0.3 is a commonly used cutoff for retaining an indicator as important in exploratory factor analysis. When the "strong" loadings set to 0.9 (Table 6), the bias ranges from about 2.3 (115%) to about 0.1 (5%). Thus, regardless of method, full (or nearly full) bias reduction is only possible when (a) all indicators are strongly related to the latent factor and (b) the strengths of those relationship are very high.

Dimension reduction results for covariate selection are summarized graphically in Figures 2 to 4. As expected, the covariate selection method tended to select fewer indicators when fewer had large loadings and more indicators were selected when more were generated with large loadings. Any differences among the four methods were relative minor in comparison to differences due to the magnitudes of factor loadings. Finally, we note that the hybrid approach performed well across all conditions.

Conclusions

Our first conclusion is that factor scores from a confirmatory factor analysis may be used to reduce the dimension of a set of manifest indicators without a detrimental loss in capacity for bias reduction. Here we underscore the point that factor analysis reduced the dimension of the indicator space from ten down to one, whereas, the other methods used either all ten indicators or used some number selected by GBN, typically between five and ten. In practice, to reduce the dimension of indicators, researchers often take sum scores of inventories that are not meant to be summed. What our results show is that it may be acceptable to use factor scores instead.

The second conclusion is the importance of working with valid indicators that truly measure the latent construct they purport to. These results perhaps may be used as a warning for researchers considering using untested and unvalidated items as proxies for latent constructs in observational study settings.

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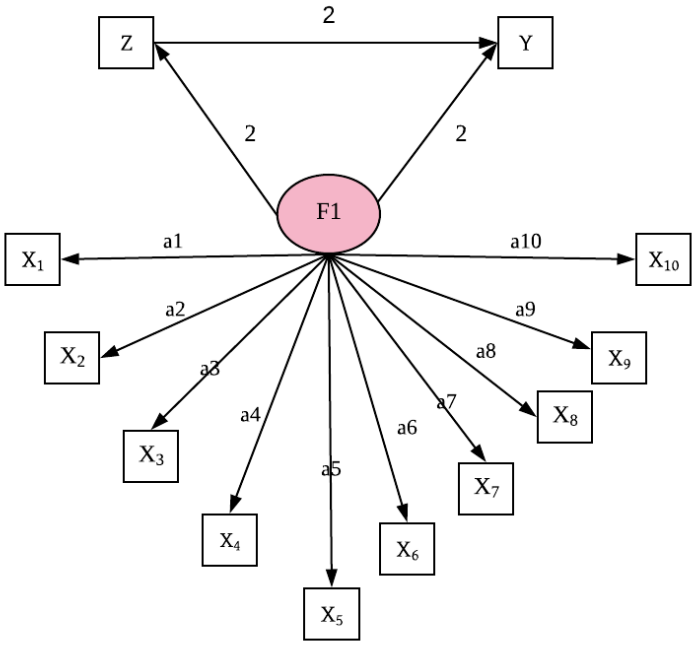

Figure 1. Data_generating process for simulation study; F1 is the latent confounder; a's are the factor loadings for each constructs.

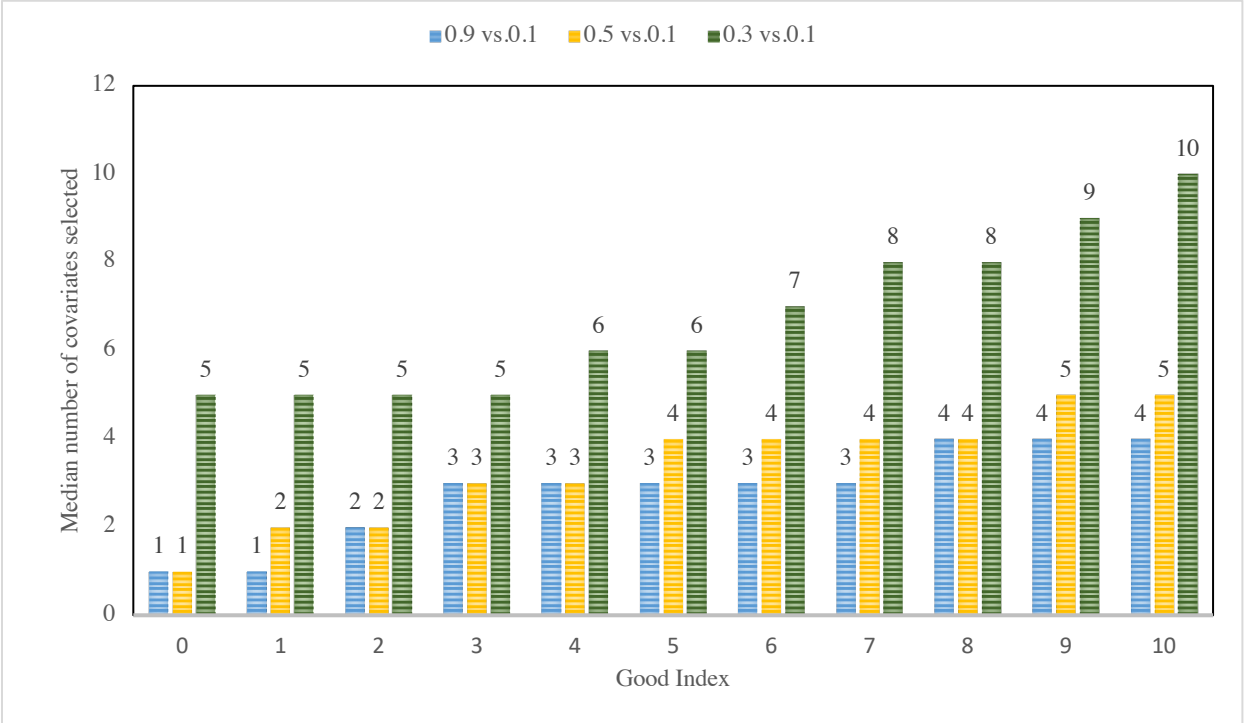

Figure 2. Median number of covariates selected when sample size n = 100; Good Index is the number of high loading items included.

Figure 3. Medium sample size for covariate selection methods when n=500; Good Index is the number of high loading items included.

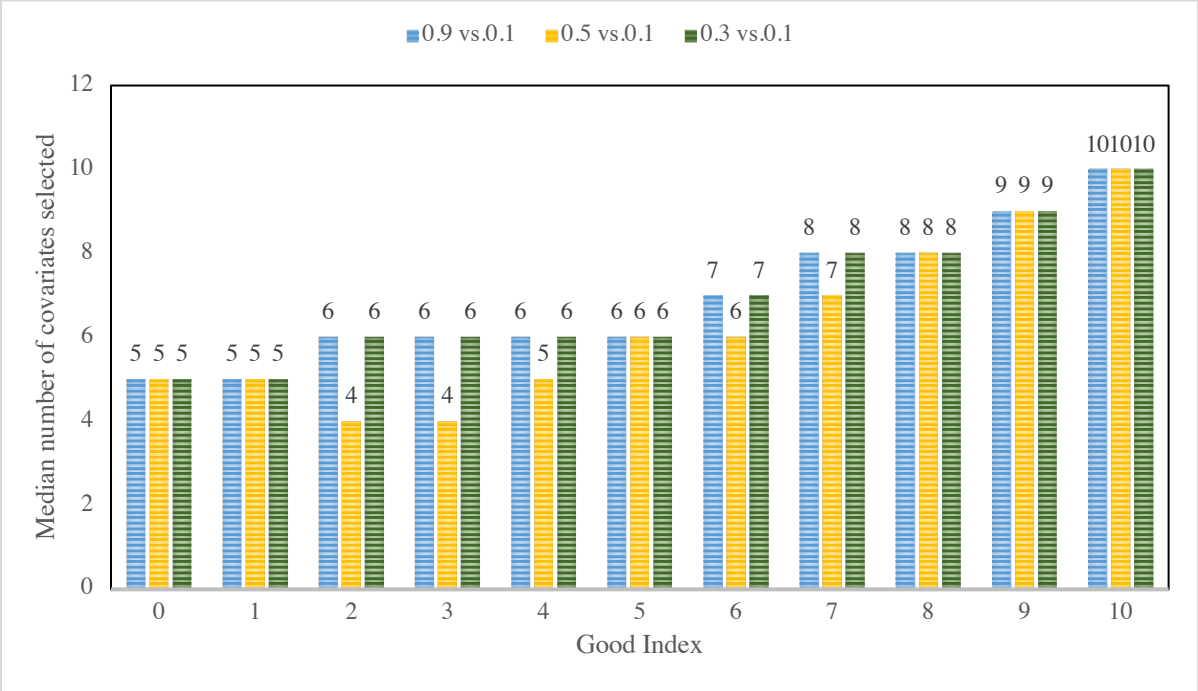

Figure 4. Medium sample size for covariate selection methods when n=1000; Good Index is the number of high loading items included.

Small Sample Size n=100

Methods	Good. Index	Bias	S.D	MSE
All	0	2.1913	0.4041	4.9635
Factor	0	2.3157	0.3627	5.4928
Select	0	2.2893	0.3724	5.3783
Hybrid	0	2.2742	0.3740	5.3106
All	1	2.1663	0.4015	4.8527
Factor	1	2.3698	0.3849	5.7626
Select	1	2.3019	0.3874	5.4471
Hybrid	1	2.2741	0.4014	5.3309
All	2	2.0594	0.3694	4.3763
Factor	2	2.3334	0.3148	5.5430
Select	2	2.2519	0.3734	5.2091
Hybrid	2	2.2168	0.3680	5.0483
All	3	1.9818	0.3800	4.0704
Factor	3	2.2891	0.3849	5.3864
Select	3	2.1758	0.3886	4.8835
Hybrid	3	2.1204	0.3783	4.6378
All	4	1.8695	0.3819	3.6393
Factor	4	2.1639	0.3755	4.8220
Select	4	2.0700	0.3879	4.4337
Hybrid	4	1.9975	0.3730	4.1280
All	5	1.8606	0.3757	3.6015
Factor	5	2.1992	0.4029	4.9974
Select	5	2.1662	0.4198	4.8668
Hybrid	5	2.0364	0.4108	4.3141
All	6	1.7851	0.4193	3.3607
Factor	6	2.0765	0.4081	4.4767
Select	6	2.0723	0.4310	4.4782
Hybrid	6	1.9136	0.4227	3.8387
All	7	1.6285	0.3387	2.7656
Factor	7	1.8816	0.4074	3.7049
Select	7	1.9407	0.3896	3.9166

Hybrid	7	1.7570	0.3678	3.2211
All	8	1.5804	0.3690	2.6326
Factor	8	1.7929	0.4134	3.3839
Select	8	1.9222	0.3867	3.8427
Hybrid	8	1.6987	0.3809	3.0293
All	9	1.5493	0.3850	2.5470
Factor	9	1.7525	0.3605	3.1999
Select	9	1.8775	0.4088	3.6903
Hybrid	9	1.6453	0.3605	2.8358
All	10	1.4072	0.3502	2.1015
Factor	10	1.6659	0.3888	2.9248
Select	10	1.7912	0.3531	3.3318
Hybrid	10	1.5421	0.3442	2.4954

Table 1: This table is the result of bias, standard deviation (S.D) and MSE of estimated treatment effect with sample size=100; Good. Index are the number of measures with factor loading=0.3 included; Good. Index=0 means factor loading=0.1; Methods are different approaches to deal with latent confounding: All indicate kitchen sink approach, Hybrid is the hybrid approach, Select is the covariate selection approach and Factor is the factor analysis approach.

Methods	**Good. Index**	**Bias**	**S.D**	**MSE**
All	0	2.1913	0.4041	4.9635
Factor	0	2.3157	0.3627	5.4928
Select	0	2.2893	0.3724	5.3783
Hybrid	0	2.2742	0.3740	5.3106
All	1	1.9373	0.3796	3.8957
Factor	1	2.3211	0.3807	5.5311
Select	1	2.0403	0.4012	4.3222
Hybrid	1	2.0318	0.3880	4.2772
All	2	1.6446	0.3521	2.8275
Factor	2	1.9786	0.3784	4.0568
Select	2	1.7376	0.3772	3.1601
Hybrid	2	1.7024	0.3567	3.0242
All	3	1.4287	0.3228	2.1444
Factor	3	1.6909	0.3926	3.0118

Select	3	1.5756	0.3412	2.5977
Hybrid	3	1.4912	0.3337	2.3340
All	4	1.3185	0.3538	1.8623
Factor	4	1.4742	0.3763	2.3133
Select	4	1.4780	0.3935	2.3378
Hybrid	4	1.3630	0.3564	1.9834
All	5	1.1684	0.3339	1.4756
Factor	5	1.2834	0.3396	1.7613
Select	5	1.3668	0.3699	2.0036
Hybrid	5	1.2184	0.3252	1.5893
All	6	1.0229	0.3096	1.1413
Factor	6	1.0939	0.2856	1.2773
Select	6	1.2139	0.3493	1.5943
Hybrid	6	1.0524	0.3079	1.2015
All	7	0.9599	0.3600	1.0496
Factor	7	1.0313	0.3527	1.1866
Select	7	1.2073	0.3984	1.6148
Hybrid	7	0.9997	0.3550	1.1242
All	8	0.8988	0.3533	0.9313
Factor	8	0.9444	0.3421	1.0078
Select	8	1.1348	0.3934	1.4410
Hybrid	8	0.9184	0.3529	0.9667
All	9	0.8713	0.3572	0.8855
Factor	9	0.9190	0.3400	0.9590
Select	9	1.1581	0.3930	1.4940
Hybrid	9	0.9060	0.3546	0.9453
All	10	0.7527	0.3420	0.6823
Factor	10	0.8093	0.3171	0.7545
Select	10	1.0609	0.3809	1.2692
Hybrid	10	0.7882	0.3386	0.7347

Table 2: This table is the result of bias, standard deviation (S.D) and MSE of estimated treatment effect with sample size=100; Good. Index are the number of measures with factor loading=0.5 included; Good. Index=0 means factor loading=0.1; Methods are different

approaches to deal with latent confounding: All indicate kitchen sink approach, Hybrid is the hybrid approach, Select is the covariate selection approach and Factor is the factor analysis approach.

Methods	Good. Index	Bias	S. D	MSE
All	0	2.1913	0.4041	4.9635
Factor	0	2.3157	0.3627	5.4928
Select	0	2.2893	0.3724	5.3783
Hybrid	0	2.2742	0.3740	5.3106
All	1	0.6491	0.3308	0.5296
Factor	1	1.7317	0.8011	3.6341
Select	1	0.6594	0.3221	0.5376
Hybrid	1	0.6606	0.3246	0.5407
All	2	0.3203	0.2592	0.1691
Factor	2	0.5173	0.2769	0.3435
Select	2	0.3308	0.2609	0.1768
Hybrid	2	0.3233	0.2505	0.1667
All	3	0.2581	0.3040	0.1581
Factor	3	0.2665	0.2961	0.1578
Select	3	0.3009	0.3153	0.1889
Hybrid	3	0.2629	0.3010	0.1588
All	4	0.1784	0.3016	0.1219
Factor	4	0.1910	0.2784	0.1132
Select	4	0.2425	0.2997	0.1477
Hybrid	4	0.1829	0.2783	0.1101
All	5	0.1348	0.2679	0.0892
Factor	5	0.1380	0.2538	0.0828
Select	5	0.2091	0.2731	0.1176
Hybrid	5	0.1347	0.2551	0.0826
All	6	0.1015	0.2940	0.0959
Factor	6	0.1145	0.2914	0.0972
Select	6	0.1495	0.3209	0.1243
Hybrid	6	0.1022	0.3000	0.0996
All	7	0.1262	0.2566	0.0811
Factor	7	0.1282	0.2544	0.0805

Select	7	0.1951	0.2738	0.1123
Hybrid	7	0.1279	0.2587	0.0826
All	8	0.0699	0.3032	0.0959
Factor	8	0.0745	0.2893	0.0884
Select	8	0.1564	0.2992	0.1131
Hybrid	8	0.0781	0.2948	0.0921
All	9	0.0794	0.3049	0.0983
Factor	9	0.0839	0.2902	0.0904
Select	9	0.1598	0.3151	0.1238
Hybrid	9	0.0881	0.3015	0.0977
All	10	0.0736	0.2758	0.0807
Factor	10	0.0854	0.2717	0.0804
Select	10	0.1508	0.3037	0.1140
Hybrid	10	0.0850	0.2814	0.0856

Table 3: This table is the result of bias, standard deviation (S.D) and MSE of estimated treatment effect with sample size=100; Good. Index are the number of measures with factor loading=0. included; Good. Index=0 means factor loading=0.1; Methods are different approaches to deal with latent confounding: All indicate kitchen sink approach, Hybrid is the hybrid approach, Select is the covariate selection approach and Factor is the factor analysis approach.

Medium Sample Size n=500

Methods	**Good. Index**	**Bias**	**S.D**	**MSE**
All	0	2.2577	0.1840	5.1308
Factor	0	2.3794	0.1909	5.6979
Select	0	2.3334	0.1847	5.4784
Hybrid	0	2.3162	0.1865	5.3992
All	1	2.1747	0.1636	4.7560
Factor	1	2.3401	0.1799	5.5081
Select	1	2.2257	0.1653	4.9806
Hybrid	1	2.2119	0.1668	4.9203
All	2	2.0511	0.1764	4.2377
Factor	2	2.2141	0.2001	4.9418
Select	2	2.1094	0.1721	4.4790
Hybrid	2	2.0866	0.1746	4.3841
All	3	1.9464	0.1772	3.8197

Factor	3	2.0624	0.1936	4.2905
Select	3	2.0021	0.1781	4.0399
Hybrid	3	1.9721	0.1745	3.9195
All	4	1.9066	0.1371	3.6536
Factor	4	1.9843	0.1496	3.9596
Select	4	1.9573	0.1374	3.8497
Hybrid	4	1.9256	0.1332	3.7255
All	5	1.7883	0.1669	3.2256
Factor	5	1.8457	0.1702	3.4354
Select	5	1.8312	0.1661	3.3808
Hybrid	5	1.8013	0.1635	3.2710
All	6	1.7235	0.1613	2.9962
Factor	6	1.7740	0.1640	3.1739
Select	6	1.7680	0.1718	3.1550
Hybrid	6	1.7322	0.1623	3.0266
All	7	1.6706	0.1435	2.8113
Factor	7	1.7245	0.1417	2.9937
Select	7	1.7112	0.1486	2.9499
Hybrid	7	1.6768	0.1421	2.8317
All	8	1.5993	0.1525	2.5807
Factor	8	1.6428	0.1516	2.7215
Select	8	1.6381	0.1636	2.7098
Hybrid	8	1.6047	0.1512	2.5977
All	9	1.5317	0.1386	2.3652
Factor	9	1.5703	0.1463	2.4869
Select	9	1.5825	0.1454	2.5254
Hybrid	9	1.5351	0.1375	2.3753
All	10	1.4934	0.1743	2.2604
Factor	10	1.5313	0.1705	2.3738
Select	10	1.5472	0.1770	2.4247
Hybrid	10	1.4954	0.1738	2.2661

Table 4: This table is the result of bias, standard deviation(S.D) and MSE of estimated treatment effect with sample size=500; Good. Index are the number of measures with factor loading=0.3 included; Good. Index=0 means factor loading=0.1; Methods are different approaches to deal with latent confounding: All indicate kitchen sink approach, Hybrid is the hybrid approach, Select is the covariate selection approach and Factor is the factor analysis approach.

Methods	Good. Index	Bias	S.D	MSE
All	0	2.2577	0.1840	5.1308
Factor	0	2.3794	0.1909	5.6979
Select	0	2.3334	0.1847	5.4784
Hybrid	0	2.3162	0.1865	5.3992
All	1	1.9158	0.1765	3.7013
Factor	1	2.1366	0.2151	4.6108
Select	1	1.9753	0.1760	3.9326
Hybrid	1	1.9475	0.1779	3.8241
All	2	1.6345	0.1795	2.7034
Factor	2	1.7492	0.2109	3.1038
Select	2	1.6764	0.1777	2.8414
Hybrid	2	1.6501	0.1798	2.7549
All	3	1.4181	0.1456	2.0320
Factor	3	1.4507	0.1467	2.1259
Select	3	1.4466	0.1461	2.1138
Hybrid	3	1.4261	0.1476	2.0553
All	4	1.2759	0.1558	1.6520
Factor	4	1.2951	0.1550	1.7010
Select	4	1.2975	0.1534	1.7068
Hybrid	4	1.2800	0.1546	1.6620
All	5	1.1520	0.1460	1.3482
Factor	5	1.1707	0.1455	1.3914
Select	5	1.1696	0.1452	1.3888
Hybrid	5	1.1571	0.1475	1.3604
All	6	1.0562	0.1439	1.1360
Factor	6	1.0806	0.1426	1.1663
Select	6	1.0730	0.1450	1.1722
Hybrid	6	1.0591	0.1442	1.1423
All	7	0.9677	0.1473	0.9578
Factor	7	0.9867	0.1458	0.9945
Select	7	0.9813	0.1482	0.9846
Hybrid	7	0.9719	0.1484	0.9665

All	8	0.8807	0.1719	0.8050
Factor	8	0.8901	0.1689	0.8205
Select	8	0.8893	0.1720	0.8202
Hybrid	8	0.8807	0.1720	0.8048
All	9	0.8366	0.1450	0.7207
Factor	9	0.8486	0.1422	0.7402
Select	9	0.8494	0.1473	0.7429
Hybrid	9	0.8374	0.1451	0.7221
All	10	0.7620	0.1416	0.6005
Factor	10	0.7701	0.1420	0.6129
Select	10	0.7764	0.1453	0.6237
Hybrid	10	0.7621	0.1415	0.6006

Table 5: This table is the result of bias, standard deviation (S.D) and MSE of estimated treatment effect with sample size=500; Good. Index are the number of measures with factor loading=0.5 included; Good. Index=0 means factor loading=0.1; Methods are different approaches to deal with latent confounding: All indicate kitchen sink approach, Hybrid is the hybrid approach, Select is the covariate selection approach and Factor is the factor analysis approach.

Methods	**Good. Index**	**Bias**	**S.D**	**MSE**
All	0	2.2577	0.1840	5.1308
Factor	0	2.3794	0.1909	5.6979
Select	0	2.3334	0.1847	5.4784
Hybrid	0	2.3162	0.1865	5.3992
All	1	0.6293	0.1387	0.4151
Factor	1	0.7844	0.3103	0.7106
Select	1	0.6392	0.1331	0.4261
Hybrid	1	0.6315	0.1359	0.4171
All	2	0.3960	0.1175	0.1705
Factor	2	0.4810	0.1417	0.2512
Select	2	0.3998	0.1206	0.1742
Hybrid	2	0.3972	0.1195	0.1720
All	3	0.2504	0.1304	0.0795
Factor	3	0.2516	0.1282	0.0796
Select	3	0.2515	0.1292	0.0798

Hybrid	3	0.2502	0.1290	0.0791
All	4	0.1742	0.1234	0.0454
Factor	4	0.1762	0.1240	0.0463
Select	4	0.1759	0.1232	0.0460
Hybrid	4	0.1750	0.1238	0.0458
All	5	0.1528	0.0903	0.0314
Factor	5	0.1531	0.0893	0.0313
Select	5	0.1551	0.0927	0.0326
Hybrid	5	0.1520	0.0900	0.0311
All	6	0.1290	0.1278	0.0328
Factor	6	0.1284	0.1262	0.0323
Select	6	0.1331	0.1264	0.0335
Hybrid	6	0.1282	0.1273	0.0325
All	7	0.0956	0.1157	0.0224
Factor	7	0.0990	0.1119	0.0222
Select	7	0.1042	0.1157	0.0241
Hybrid	7	0.0954	0.1140	0.0220
All	8	0.0871	0.1129	0.0202
Factor	8	0.0877	0.1099	0.0197
Select	8	0.0995	0.1165	0.0233
Hybrid	8	0.0870	0.1123	0.0201
All	9	0.0774	0.1252	0.0215
Factor	9	0.0779	0.1237	0.0212
Select	9	0.0906	0.1232	0.0232
Hybrid	9	0.0772	0.1241	0.0212
All	10	0.0967	0.1144	0.0223
Factor	10	0.0965	0.1148	0.0223
Select	10	0.1113	0.1157	0.0256
Hybrid	10	0.0968	0.1145	0.0224

Table 6: This table is the result of bias, standard deviation(S.D) and MSE of estimated treatment effect with sample size=500; Good. Index are the number of measures with factor loading=0.9 included; Good. Index=0 means factor loading=0.1; Methods are different approaches to deal with latent confounding: All indicate kitchen sink approach, Hybrid is the hybrid approach, Select is the covariate selection approach and Factor is the factor analysis approach.

Large Sample Size n=1000

Methods	Good. Index	Bias	S.D	MSE
All	0	2.3002	0.1137	5.3036
Factor	0	2.4036	0.1159	5.7908
Select	0	2.3560	0.1189	5.5647
Hybrid	0	2.3358	0.1165	5.4694
All	1	2.1881	0.1235	4.8030
Factor	1	2.3005	0.1473	5.3139
Select	1	2.2254	0.1273	4.9685
Hybrid	1	2.2073	0.1265	4.8882
All	2	2.0627	0.1410	4.2743
Factor	2	2.1365	0.1399	4.5842
Select	2	2.0990	0.1406	4.4255
Hybrid	2	2.0752	0.1393	4.3258
All	3	1.9535	0.1221	3.8309
Factor	3	2.0135	0.1279	4.0704
Select	3	1.9899	0.1244	3.9750
Hybrid	3	1.9667	0.1232	3.8831
All	4	1.8915	0.1155	3.5910
Factor	4	1.9292	0.1209	3.7362
Select	4	1.9207	0.1154	3.7024
Hybrid	4	1.8986	0.1157	3.6180
All	5	1.8081	0.1035	3.2798
Factor	5	1.8394	0.1077	3.3947
Select	5	1.8323	0.1067	3.3687
Hybrid	5	1.8128	0.1045	3.2970
All	6	1.7441	0.1272	3.0580
Factor	6	1.7702	0.1298	3.1503
Select	6	1.7655	0.1270	3.1331
Hybrid	6	1.7481	0.1265	3.0718
All	7	1.6672	0.1032	2.7901
Factor	7	1.6905	0.1063	2.8691
Select	7	1.6819	0.1035	2.8393

Hybrid	7	1.6698	0.1039	2.7988
All	8	1.6045	0.1166	2.5879
Factor	8	1.6242	0.1166	2.6516
Select	8	1.6140	0.1179	2.6187
Hybrid	8	1.6052	0.1168	2.5903
All	9	1.5483	0.1054	2.4082
Factor	9	1.5702	0.1051	2.4765
Select	9	1.5554	0.1079	2.4309
Hybrid	9	1.5484	0.1053	2.4085
All	10	1.4912	0.1184	2.2376
Factor	10	1.5063	0.1200	2.2831
Select	10	1.4942	0.1193	2.2468
Hybrid	10	1.4912	0.1184	2.2376

Table 7: This table is the result of bias, standard deviation(S.D) and MSE of estimated treatment effect with sample size=1000; Good. Index are the number of measures with factor loading=0.3 included; Good. Index=0 means factor loading=0.1; Methods are different approaches to deal with latent confounding: All indicate kitchen sink approach, Hybrid is the hybrid approach, Select is the covariate selection approach and Factor is the factor analysis approach.

Methods	**Good. Index**	**Bias**	**S.D**	**MSE**
All	0	2.3002	0.1137	5.3036
Factor	0	2.4036	0.1159	5.7908
Select	0	2.3560	0.1189	5.5647
Hybrid	0	2.3358	0.1165	5.4694
All	1	1.8832	0.1331	3.5639
Factor	1	1.9996	0.1865	4.0328
Select	1	1.9226	0.1307	3.7131
Hybrid	1	1.8963	0.1318	3.6133
All	2	1.6529	0.1110	2.7442
Factor	2	1.7032	0.1270	2.9169
Select	2	1.6834	0.1141	2.8469
Hybrid	2	1.6587	0.1118	2.7637
All	3	1.4366	0.1031	2.0744
Factor	3	1.4656	0.1029	2.1293

Select	3	1.4627	0.1045	2.1502
Hybrid	3	1.4428	0.1027	2.0921
All	4	1.2761	0.1069	1.6396
Factor	4	1.2863	0.1058	1.6656
Select	4	1.2957	0.1062	1.6901
Hybrid	4	1.2798	0.1068	1.6491
All	5	1.1776	0.1023	1.3970
Factor	5	1.1874	0.1002	1.4198
Select	5	1.1923	0.1020	1.4318
Hybrid	5	1.1792	0.1022	1.4009
All	6	1.0478	0.0972	1.1072
Factor	6	1.0536	0.0974	1.1194
Select	6	1.0567	0.0968	1.1258
Hybrid	6	1.0485	0.0969	1.1086
All	7	0.9645	0.1111	0.9424
Factor	7	0.9696	0.1113	0.9524
Select	7	0.9705	0.1102	0.9539
Hybrid	7	0.9654	0.1108	0.9441
All	8	0.8940	0.1069	0.8106
Factor	8	0.8991	0.1066	0.8196
Select	8	0.8974	0.1072	0.8168
Hybrid	8	0.8941	0.1072	0.8108
All	9	0.8382	0.1026	0.7131
Factor	9	0.8430	0.1032	0.7212
Select	9	0.8391	0.1032	0.7147
Hybrid	9	0.8382	0.1026	0.7131
All	10	0.7815	0.0888	0.6185
Factor	10	0.7872	0.0899	0.6277
Select	10	0.7815	0.0888	0.6185
Hybrid	10	0.7815	0.0888	0.6185

Table 8: This table is the result of bias, standard deviation(S.D) and MSE of estimated treatment effect with sample size=1000; Good. Index are the number of measures with factor loading=0.5 included; Good. Index=0 means factor loading=0.1; Methods are different approaches to deal with latent confounding: All indicate kitchen sink approach, Hybrid is the hybrid approach, Select is the covariate selection approach and Factor is the factor analysis approach.

Methods	Good. Index	Bias	SD	MSE
All	0	2.3002	0.1137	5.3036
Factor	0	2.4036	0.1159	5.7908
Select	0	2.3560	0.1189	5.5647
Hybrid	0	2.3358	0.1165	5.4694
All	1	0.6370	0.1141	0.4187
Factor	1	0.6839	0.1590	0.4928
Select	1	0.6416	0.1145	0.4246
Hybrid	1	0.6367	0.1149	0.4185
All	2	0.3798	0.0944	0.1531
Factor	2	0.4168	0.1079	0.1853
Select	2	0.3832	0.0940	0.1556
Hybrid	2	0.3808	0.0939	0.1537
All	3	0.2516	0.0945	0.0721
Factor	3	0.2533	0.0935	0.0728
Select	3	0.2529	0.0937	0.0726
Hybrid	3	0.2514	0.0935	0.0719
All	4	0.1970	0.0876	0.0464
Factor	4	0.1996	0.0866	0.0472
Select	4	0.1989	0.0872	0.0471
Hybrid	4	0.1980	0.0875	0.0468
All	5	0.1863	0.0867	0.0422
Factor	5	0.1893	0.0872	0.0434
Select	5	0.1872	0.0871	0.0426
Hybrid	5	0.1873	0.0867	0.0425
All	6	0.1374	0.0872	0.0264
Factor	6	0.1380	0.0888	0.0268
Select	6	0.1380	0.0878	0.0267
Hybrid	6	0.1373	0.0876	0.0265
All	7	0.1106	0.0815	0.0188
Factor	7	0.1115	0.0802	0.0188
Select	7	0.1117	0.0814	0.0190
Hybrid	7	0.1104	0.0811	0.0187
All	8	0.1159	0.0708	0.0184

Factor	8	0.1144	0.0713	0.0181
Select	8	0.1187	0.0710	0.0191
Hybrid	8	0.1155	0.0706	0.0183
All	9	0.0978	0.0870	0.0171
Factor	9	0.0990	0.0864	0.0172
Select	9	0.1028	0.0875	0.0181
Hybrid	9	0.0980	0.0867	0.0170
All	10	0.0879	0.0851	0.0149
Factor	10	0.0882	0.0845	0.0148
Select	10	0.0939	0.0854	0.0160
Hybrid	10	0.0878	0.0851	0.0149

Table 9: This table is the result of bias, standard deviation (S.D) and MSE of estimated treatment effect with sample size=1000; Good.Index are the number of measures with factor loading=0.9 included; Good. Index=0 means factor loading=0.1; Methods are different approaches to deal with latent confounding: All indicate kitchen sink approach, Hybrid is the hybrid approach, Select is the covariate selection approach and Factor is the factor analysis approach.