Modeling Intervention Effects on Social Networks in Education Research

Tracy M. Sweet and Brian W. Junker

Carnegie Mellon University

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Outline

1 Social Networks in Education Research

2 Hierarchical Network Models
   • Hierarchical Latent Space Models For Observational Data
   • Hierarchical Latent Space Models for Interventions

3 Future Work
What is a Social Network?

A social network is a directed or undirected graph consisting of individuals (vertices, nodes) and the relationships (edges, ties) among them.

Figure: RWLA Advice Network
What is a Social Network?

A social network is a directed or undirected graph consisting of individuals (vertices, nodes) and the relationships (edges, ties) among them.

▶ A social network among \( n \) individuals can be represented by an \( n \times n \) edge matrix \( Y \),

\[
Y = \begin{bmatrix}
Y_{11} & Y_{12} & \cdots & Y_{1n} \\
\vdots & \vdots & \ddots & \vdots \\
Y_{n1} & Y_{n2} & \cdots & Y_{nn}
\end{bmatrix}
\]

▶ \( Y_{ij} \) = the tie from person \( i \) to \( j \)

▶ \( Y_{ij} \) may be discrete (absence/presence) or continuous (strength of tie)

Figure: RWLA Advice Network
Social Networks of Teachers

► Association between existing network structure and intervention implementation
  ▶ Well-connected teachers were more likely to change their teaching practices to align with initiative practices (Penuel et al., 2006)
  ▶ Principal centrality related to teacher willingness to invest in change (Moolenaar et al., 2010)

► Association between network structure and student outcomes
  ▶ Teacher reported access to resources associated with student achievement (Leana and Pil, 2006)

► Intervention may influence network structure
  ▶ Teachers in schools involved in school-wide initiatives were more connected to others (Weinbaum et al., 2008)
Social Network Statistical Modeling Families

1. Exponential Random Graph Model (Wasserman and Pattison, 1996)
   - Probability of observed network is a function of covariates and network statistics

2. Latent Space Model (Hoff et al., 2002)
   - Probability of a tie determined by covariates and proximity of teacher’s latent positions in social space

3. Mixed Membership Stochastic Block Model (Airoldi et al., 2008)
   - Probability of a tie determined by group membership of each actor which are context specific
Existing Methodology is Inadequate

**Summary Statistics**

- Represent each individual or entire school network with one statistic
- Imbed statistic into a linear model
- Ignore much of the network structure

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**Figure**: Advice Networks from Pitts and Spillane (2009)
Existing Methodology is Inadequate

Social Network Models
- Model the entire school network structure
- Model only one network at a time
- Fit separate models for each school

Figure: Advice Networks from Pitts and Spillane (2009)
Existing Methodology is Inadequate

Neither Method...
- models more than one full network at a time
- allows us to compare treated and untreated networks

Figure: Advice Networks from Pitts and Spillane (2009)
Current Education Research Work
Chicago Study (Spillane, 2009, IES-funded)

- 3 year study in a number of urban district elementary schools

- Treatment schools will implement leadership training/collaboration routines
  - Promote discussion and feedback about teacher quality
  - Involve teachers in observing other teachers
  - Restructures relationships between principals and staff

- Friendship and advice-seeking data will be collected from all staff

We need methods that can...
- model the entire sample of networks
- determine treatment effects
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The Hierarchical Network Model (HNM) Framework

Let $Y_{ijk}$ is the tie from $i$ to $j$ in network $k$.

\[
P(Y|X, \Theta) = \prod_{k=1}^{K} P(Y_k|X_k, \Theta_k = (\theta_{1k}, \ldots, \theta_{pk}))
\]

\[
\theta_{1k} \sim f_{1k}(\eta_{1k}, \tau_{1k})
\]

\[
\vdots
\]

\[
\theta_{pk} \sim f_{pk}(\eta_{pk}, \tau_{pk})
\]

where $P(Y_k|X_k, \Theta_k)$ is a statistical social network model for network $k$.
The Hierarchical Network Model (HNM) Framework

The Hierarchical Network Model (HNM) Framework
The Hierarchical Latent Space Model (HLSM) For Observational Data

\[ P(Y|X, \beta, Z) = \prod_k \prod_{i \neq j} P(Y_{ijk}|Z_{ik}, Z_{jk}, X_{ijk}, \beta_k) \]

\[ \text{logit}(P(Y_{ijk} = 1)) = \beta_k X_{ijk} - |Z_{ik} - Z_{jk}| \]

\[ Z_{ik} \sim \text{MVN}(\mu_k, \Sigma_k) \]

\[ \beta_{0k} \sim N(\nu_{0k}, \sigma_{0k}) \]

\[ \vdots \]

\[ \beta_{pk} \sim N(\nu_{pk}, \sigma_{pk}) \]

- \( X_{ijk} \) is an individual or tie-specific covariate for network \( k \)
- \( Z_{ik} \) is the latent space position for individual \( i \) in network \( k \)
Fitting the HLSM For Observational Data

- Teacher Advice Network Data (Pitts and Spillane, 2009)
- Binary, Undirected Ties
- 15 Elementary schools
- 2D latent positions
Fitting the HLSM For Observational Data

- Teacher Advice Network Data (Pitts and Spillane, 2009)
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Model

\[
\text{logit}(P(Y_{ijk} = 1)) = \beta_{0k} + \beta_{1k}X_{ijk} - |Z_{ik} - Z_{jk}|
\]

\[
Z_{ik} \sim MVN \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}
\]

\[
\beta_{0k} \sim N(0, 100)
\]

\[
\beta_{1k} \sim N(0, 100)
\]

\(X_{ijk}\) is the indicator that teacher \(i\) and \(j\) in network \(k\) teach the same grade

- Coded model fitting algorithm (MCMC) in R
Fitting the HLSM For Observational Data

Figure: Effect of Teaching the Same Grade
A Hierarchical Latent Space Model (HLSM) for Interventions

\[
P(Y|X, \beta, Z, T, \alpha) = \prod_k \prod_{i \neq j} P(Y_{ijk}|Z_{ik}, Z_{jk}, X_{ijk}, \beta_k, T, \alpha)
\]

\[
\text{logit}(P(Y_{ijk} = 1)) = \beta_k X_{ijk} - |Z_{ik} - Z_{jk}| + \alpha T_k
\]

\[
Z_{ik} \sim \text{MVN}(\mu_k, \Sigma_k)
\]

\[
\beta_{0k} \sim N(\nu_{0k}, \sigma_{0k})
\]

\[
\vdots
\]

\[
\beta_{pk} \sim N(\nu_{pk}, \sigma_{pk})
\]

\[
\alpha \sim N(\eta, \tau)
\]

- \(X_{ijk}\) is an individual or tie-specific covariate for network \(k\)
- \(Z_{ik}\) is the latent space position for individual \(i\) in network \(k\)
- \(T_k\) is the treatment assigned to network \(k\)

\*
*This is one of several possible HLSM’s for interventions.*
Fitting the HLSM for Interventions

Simulated Data

- Binary, Undirected Ties
- 10 teachers per school
- 20 schools
- 2D latent positions

Data Generating Model

$$\text{logit}(P(Y_{ijk} = 1)) = 2 + 4X_{ijk} - |Z_{ik} - Z_{jk}| + \alpha T_k$$

Treatment Effect: $\alpha = 3, 2, 1, 0.5$

$X_{ijk}$ is the indicator that teacher $i$ and $j$ in network $k$ teach the same grade
Treatment Effect: $\alpha = 3$
Treatment Effect: $\alpha = 2$
Treatment Effect: $\alpha = 1$
Treatment Effect: $\alpha = 0.5$
Fitting the HLSM for Interventions

Fitted Model

\[
\text{logit}(P(Y_{ijk} = 1)) = \beta_0 + \beta_{1k} X_{ijk} - |Z_{ik} - Z_{jk}| + \alpha T_k
\]

\(Z_{ik} \sim \text{MVN}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}\right)\)

\(\beta_0 \sim N(0, 100)\)

\(\beta_{1k} \sim N(0, 100)\)

\(\alpha \sim N(0, 100)\)

\(X_{ijk}\) is the indicator that teacher \(i\) and \(j\) in network \(k\) teach the same grade

- Coded model fitting algorithm (MCMC) in R
MCMC Posterior Densities for Treatment Effect ($\alpha$)

$\alpha = 3$

$\alpha = 2$

$\alpha = 1$

$\alpha = 0.5$
Summary

- Introduced Hierarchical Network Models (HNMs) and gave several examples
  - The Hierarchical Latent Space Model (HLSM) for Observational Data
    - Model fit with real data to illustrate utility
  - A Hierarchical Latent Space Model (HLSM) for Interventions
    - Model fit with simulated data to assess parameter recovery
    - Treatment effects recovered for $\alpha = 3, 2, 1$ but not for $\alpha = 0.5$
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Future Work

- Develop new models and model-fitting algorithms
  - Hierarchical Mixed Membership Stochastic Block Models (HMMSBM) for Observational Data
  - Hierarchical Mixed Membership Stochastic Block Models (HMMSBM) for Interventions

- Investigate operational characteristics
  *When can we accurately recover treatment effects?*
  - Power analyses
  - Missing data
  - Model selection and specification


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  \textsuperscript{1} Department of Statistics, Carnegie Mellon University
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  \textsuperscript{3} Department of Psychology, Carnegie Mellon University
  \textsuperscript{4} Machine Learning Department, Carnegie Mellon University

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Contact Information: tsweet@stat.cmu.edu
Why Latent Space Models?

- Linear model analogies
  - Generalization similar to HLM framework
  - Interpretation similar to HLM
- Discrete or continuous tie data
- Clustering feasibility

- Why NOT ERGM’s?
  - Binary ties only
  - Difficult to explain to non-stat audiences
  - Unintuitive generalization for interventions
Chicago Study (Spillane, 2009, IES-funded)

- 3 year study in urban district elementary schools
- Treatment schools will implement leadership training/collaboration routines
- Friendship and advice-seeking data will be collected from all staff
- Social Network Hypotheses:
  - Redistribution of leaders among subgroups
  - Increase in teacher collaboration
  - Overall increase in advice seeking
  - Teachers seeking advice outside their grade level


- Observational study
- Students in 140 schools
- Friendship data: type of interaction and frequency
- Large amount of background data
Network Level Intervention Simulation Results

<table>
<thead>
<tr>
<th>True $\alpha$</th>
<th>Posterior Mode</th>
<th>95% Credible Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3.04</td>
<td>(2.36, 3.68)</td>
</tr>
<tr>
<td>2</td>
<td>1.60</td>
<td>(0.99, 2.23)</td>
</tr>
<tr>
<td>1</td>
<td>0.78</td>
<td>(0.045, 1.35)</td>
</tr>
<tr>
<td>0.5</td>
<td>0.58</td>
<td>(-0.10, 1.08)</td>
</tr>
</tbody>
</table>
MMSBM for Network-Level Intervention

Example: Intervention Affects Subgroups

\[ Y_{ijk} \sim Bernoulli(Z'_{ijk} \rightarrow B_k Z_{jik} \leftarrow) \]
\[ Z_{ijk} \rightarrow \sim Multinomial(1, \theta_{ik}) \]
\[ Z_{jik} \leftarrow \sim Multinomial(1, \theta_{jk}) \]
\[ \theta_{ik} \sim Dirichlet(\gamma_k) \]
\[ B_{mnk} \sim Beta(a, b) \]
\[ \gamma_k = \gamma_0 + \alpha T_k \]

- \( B_k \) is the group-group compatibility matrix for network \( k \)
- \( Z_{i \rightarrow j, k} \) is the latent sender group membership indicator for \( i \rightarrow j \) in \( k \)
- \( Z_{i \leftarrow j, k} \) is the latent receiver group membership indicator for \( j \rightarrow i \) in \( k \)
- \( \theta_k \) is the set of mixed membership vectors for actors in network \( k \)
- \( T_k \) is the treatment assigned to network \( k \)