Modeling Intervention Effects on Subgroup Structure Across Social Networks

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Outline

1. Introduction to Social Networks in Education Research

2. The Hierarchical Network Modeling (HNM) Framework

3. The Hierarchical Mixed Membership Stochastic Blockmodel (HMMSBM)
   - What is Mixed Membership?
   - HMMSBM for an Intervention

4. Conclusions and Future Work
What is a Social Network?

A social network is a set of relations or ties among individuals.

- A network is represented by its actors and ties.
- A matrix $Y$ represents network, $Y_{ij} =$ the tie from actor $i$ to $j$.

$$
Y = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 & 0 \\
\end{bmatrix},
$$

- $Y_{ij}$ can be discrete or continuous, with direction or without.

Figure: Reading Advice Network
Social Networks of Teachers

- Teaching the same grade and being in a leadership role promotes advice ties more than individual characteristics (Spillane et al., 2011)

- Existing network structure may influence intervention implementation
  - Forming new ties related to an initiative predicted by being in the same subgroup/same grade (Penuel et al., 2010)
  - Principal centrality related to teacher willingness to invest in change (Moolenaar et al., 2010)
  - Dense subgroups more involved with intervention than sparse subgroups (Daly et al., 2010)

- An intervention may influence network structure
  - Teachers in schools involved in school-wide initiatives were more connected to others (Weinbaum et al., 2008)
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Motivation

We want methods that can

- model the full structure of each network
- model the full sample of networks simultaneously
- compare treated and untreated networks

Figure: Advice Networks from Pitts and Spillane (2009)
Hierarchical Network Models

\[ \Theta_1 \rightarrow \Theta_2 \rightarrow \cdots \rightarrow \Theta_{K-1} \rightarrow \Theta_K \rightarrow \cdots \]

Sweet, Thomas & Junker (CMU)
Hierarchical Network Models

\[ P(\mathbb{Y}|\mathbb{X}, \Theta) = \prod_{k=1}^{K} P(Y_k|X_k = (X_{1k}, \ldots, X_{Pk}), \Theta_k = (\theta_{1k}, \ldots, \theta_{Qk})) \]

\[ (\Theta_1, \ldots, \Theta_K) \sim F(\Theta_1, \ldots, \Theta_K|W_1, \ldots, W_K, \psi) , \]

- \( P(Y_k|X_k, \Theta_k) \) is a model for a single network \( Y_k \) with covariates \( X_k \) and parameters \( \Theta_k \)
- \( W_k \) can model a variety of dependence assumptions across networks
- \( \psi \) may specify additional hierarchical structure on \( \Theta \)
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Motivation

We want methods that can

- model the full structure of each network
- model the full sample of networks simultaneously
- compare treated and untreated networks
- accommodate subgroup structure

\[
\begin{bmatrix}
* & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & * & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & * & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & * & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & * & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & * & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & * & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & *
\end{bmatrix}
\]
What is Mixed Membership?

Typical blockmodels assume each individual belongs to one subgroup.

Mixed membership allows individuals to belong to multiple subgroups.
What is a Mixed Membership Network?

Network from Blockmodel

Network from MM Blockmodel
The Hierarchical Mixed Membership Stochastic Blockmodel (HMMSBM)

\[ P(Y_{ijk} = 1) = B_k[S_{ijk}, R_{jik}] \]
\[ P(S_{ijk} = s) = \theta_{isk} \]
\[ P(R_{jik} = r) = \theta_{jrk} \]

- \( B_k[S, R] \) is the probability of a tie from group S to group R in network k
- \( S_{ijk} \) is the group membership of person i when sending a tie to person j
- \( R_{jik} \) is the group membership of person j when receiving a tie from person i
- \( \theta_{isk} \) is the probability that person i in network k belongs to group s

\[ \theta_{ik} \sim Dirichlet(\lambda_k) \]

Small values of \( \lambda_k \) generate extreme the membership probabilities
A HMMSBM for an Intervention

Suppose an intervention is hypothesized to change isolated subgroups...

\[ P(Y_{ijk} = 1) = B_k[S_{ijk}, R_{jik}] \]
\[ P(S_{ijk} = s) = \theta_{isk} \]
\[ P(R_{jik} = r) = \theta_{jrk} \]

\[ \theta_{ik} \sim \text{Dirichlet}(\lambda_k) \]
\[ \lambda_k = \lambda_0 + T_k(\vec{1} - \lambda_0)(1 - \alpha) \]

▶ \( T_k \) is treatment indicator
▶ \( \vec{1} \) is \((1, \ldots, 1)\) with length equal to number of groups
▶ \( \alpha \) is the treatment effect
▶ \( \lambda_0 \) generates the control group’s membership probabilities
Simulated Data from 20 Networks of Size 20; $\alpha = 0.53$

$$\theta_{ik} \sim \text{Dirichlet}(\lambda_k)$$

$$\lambda_k = \lambda_0 + T_k(\vec{1} - \lambda_0)(1 - \alpha)$$

$$\lambda_k = 0.05 + 0.45 T_k$$
A HMMSBM for an Intervention

Fitted Model

<table>
<thead>
<tr>
<th>Model Statement</th>
<th>Mathematical Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y_{ijk} \sim Bernoulli(B_k[S_{ijk}, R_{jik}]) )</td>
<td></td>
</tr>
<tr>
<td>( S_{ijk} \sim Multinomial(\theta_{ik}, 1) )</td>
<td></td>
</tr>
<tr>
<td>( R_{jik} \sim Multinomial(\theta_{jk}, 1) )</td>
<td></td>
</tr>
<tr>
<td>( \theta_{ik} \sim Dirichlet(\lambda_k) )</td>
<td></td>
</tr>
<tr>
<td>( B_{\ell k} \sim Beta(3, 1) )</td>
<td></td>
</tr>
<tr>
<td>( B_{\ell mk} \sim Beta(1, 10), \ell \neq m )</td>
<td></td>
</tr>
<tr>
<td>( \lambda_k = \lambda_0 + T_k(\overrightarrow{1} - \lambda_0)(1 - \alpha) )</td>
<td></td>
</tr>
<tr>
<td>( \alpha \sim Uniform(0, 1) )</td>
<td></td>
</tr>
</tbody>
</table>

- Model fit using MCMC algorithm coded in R
- \( \lambda_0 = 0.05 \)
- 4 subgroups assumed a priori
Treatment Effect Parameter ($\alpha$) Recovery
Tie Probability Recovery
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Conclusions

▶ Introduce the HNM Framework
  ▶ A flexible framework for modeling ensembles of social networks with a variety of dependence assumptions
  ▶ Incorporates entire network structure through choice of statistical social network model
  ▶ Can estimate treatment effects
  ▶ Accommodates subgroup structure

▶ Introduce the HMMSBM
  ▶ A type of HNM for subgroup structure
  ▶ Can estimate treatment effects
  ▶ Model fit with simulated data demonstrates feasibility
  ▶ Treatment effect parameter recovery is accurate
  ▶ Tie probability recovery is most accurate for networks with block structure
References


Future Work

- HMMSBM
  - Extend HMMSBM for valued ties
  - Apply HMMSBM to real datasets
  - Investigate model selection and specification
- Power Analysis for HNMs
  - Analytical sample size calculations (number of ties and number of )
  - Simulation studies to assess analytical results

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