Development of a Comprehensive Intervention to Improve Children’s Understanding of Math Equivalence
Caroline E. Byrd, Nicole M. McNeil, Heather Brletic-Shiple, and Julia M. Matthews, University of Notre Dame

**Abstract**

Most elementary school children struggle to understand mathematical equivalence. Research has shown that children’s understanding can be improved by modifying arithmetic practice to be less narrow and more in line with the underlying concepts. However, researchers have yet to develop a comprehensive intervention that produces mastery-level understanding in most children. Our goal is to develop such an intervention.

**Background**

Mathematical equivalence is the relation between two quantities that are interchangeable. It is widely regarded as one of the most important concepts for developing children’s algebraic thinking.

Unfortunately, most children (ages 7-11) struggle to understand this fundamental concept. We argue that difficulties are due to children’s overly narrow experience with arithmetic in school.

Our account suggests that understanding of math equivalence can be improved by modifying arithmetic practice to be less narrow and more in line with the underlying concepts. Indeed, interventions targeting various components in isolation have improved children’s understanding. However, none of the component interventions have been sufficient on their own to produce a mastery-level understanding.

Our goal is to draw on and extend the success of these component interventions (described below) to develop a comprehensive intervention that produces a mastery-level understanding of math equivalence (at least 75% accuracy when solving and encoding equations, and defining the equal sign relationally).

Our intervention includes: (a) practice with “=” outside of an arithmetic context (e.g., 7 = 7), (b) nontraditional arithmetic practice that involves nontraditional problem formats (e.g., __ = 8 + 4), organizing problems by equivalent values (e.g., 5 + 6, 2 + 4), and using relational phrases such as “is equal to” in place of the equal sign, (c) concreteness fading exercises to strengthen the mappings between real-world contexts (e.g., balancing a scale) and formal symbols, and (d) activities requiring comparison and explanation of different problem formats (e.g., 7 = 5 + 2 and 5 + 2 = 7) and different problem-solving strategies (e.g., correct compared to incorrect).

**Method**

Our intervention includes: (a) practice with “=” outside of an arithmetic context (e.g., 7 = 7), (b) nontraditional arithmetic practice that involves nontraditional problem formats (e.g., __ = 8 + 4), organizing problems by equivalent values (e.g., 5 + 6, 2 + 4), and using relational phrases such as “is equal to” in place of the equal sign, (c) concreteness fading exercises to strengthen the mappings between real-world contexts (e.g., balancing a scale) and formal symbols, and (d) activities requiring comparison and explanation of different problem formats (e.g., 7 = 5 + 2 and 5 + 2 = 7) and different problem-solving strategies (e.g., correct compared to incorrect).

**Goal**

Our goal is to develop such an intervention.

**Assessments**

- Equation solving—Solve math equivalence problems (e.g., 1 + 5 = __ + 2)
- Equation encoding—Reconstruct math equivalence problems after viewing for 5 sec.
- Equal sign understanding—Define the equal sign relationally

**Year 1**

Goal

- Use children’s performance and tutor feedback to determine which components to include

Method

- 23 second graders (ages 7-8; 13 boys, 10 girls; 70% African American or black, 17% white, 9% other, 4% Hispanic or Latino)
- One-on-one tutoring: 34 15-minute sessions
- Assessed understanding of math equivalence with a pretest, mid-test, and posttest

**Year 2**

Goal

- Work with teacher collaborator to optimize the intervention for use in a classroom setting

Method

- 23 second graders (ages 7-8; 10 boys, 13 girls; 83% white, 13% African American or black, 2% Asian, 2% Hispanic or Latino)
- Classroom setting; 32 15-minute sessions
- Assessed understanding of math equivalence with a pretest, mid-test, and posttest

**Assessments**

- Same as Year 1

**Summary and Conclusions**

- Results from one-on-one tutoring sessions in Year 1 and the classroom setting in Year 2 provide evidence that our comprehensive intervention is leading children to construct a mastery-level understanding of math equivalence
- In the 2013-2014 school year, we will further test the effectiveness and feasibility of our intervention in a randomized, classroom-based pilot study

**Grant**

R305A110198